Effects of Current and Voltage Uncertainty on Triple Langmuir Probe Measurements

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Triple Langmuir probes are a widely utilized plasma diagnostic tool used to infer electron temperature and number density without using a voltage sweep, making them ideal for fast - changing, transient plasmas. Previous work showed that triple probes are sensitive to model assumptions, but a broader examination on the effects of current and voltage uncertainty has yet to be quantified. This present work aims to establish a quantitative baseline which investigates how uncertainty in the measured current and applied voltage propagates into uncertainty in the inferred plasma properties. In this work, the sensitivity of triple probe plasma parameters is examined. While traditional probe analysis assumes that the voltages stay constant throughout a measurement, time varying plasma behaviors may prevent the voltage from maintaining a steady-state through the pulse. By implementing a triple probe current model, this present work aims to quantify how systemic variations in the applied voltage propagate into the inferred electron temperature and number density. The results indicate that the electron temperatures demonstrate nonlinear sensitivity to voltage deviations, while the error in the number densities are resulting from violation of the ion saturation current assumption or electron temperature deviation.

Nomenclature

A_p - Probe Area [m^2]
e - Fundamental Electron Charge [q]
I_e - Electron Current [A]
I_i - Ion Current [A]
I_p - Probe Current [A]
k_B - Boltzmann's Constant
m_e - Electron Mass [kg]
m_i - Ion Mass [kg]
n_e - Electron Number Density [m^{-3}]
T_e - Electron Temperature [eV, deg K]
V - Voltage [V]
V_{d2} - Externally Applied Voltage Between probes 1-2 [V]
V_{d3} - Externally Applied Voltage Between probes 1-3 [V]
ϵ - Uncertainty Level [%]
ϕ_p - Plasma Potential [V]
ϕ_f - Floating Potential [V]
σ - Standard Deviation

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I. Introduction

Triple Langmuir probes are commonly used plasma diagnostic tools due to their relative simplicity and versatility. Since the initial development of the Langmuir probe [1], several probe configurations have been introduced: the single probe, double probe, and triple probe. Each configuration is used to determine electron temperature, number density, and plasma potential - all properties which can be extracted from a current-voltage (I-V) diagram. The triple probe configuration [2], which is the focus of this paper, involves inserting three probe tips with a fixed voltage difference between the probe tips into the plasma. The probes are then able to collect current, and with known applied voltages, are able to infer electron temperature and number density of the transient plasma.

Even though triple probes have their advantages, they rely heavily on the assumption the plasma is Maxwellian. In previous work, Polzin *et al.* [3] numerically simulated probe current collection in non-equilibrium plasmas and evaluated the results using standard triple probe theory, which assumes a Maxwellian plasma. The study looked at two-temperature Maxwellian distributions and plasmas with suprathermal electron beams. In both of the cases, deviations of the inferred plasma parameters from the input plasma parameters were observed. These results demonstrate that triple probes measurements are highly sensitive to deviations from equilibrium conditions.

While previous work has established sensitivity to distribution function variation, the impact of measurement uncertainty has yet to be quantified. Most studies assume ideal conditions which include noiseless currents and stable applied voltages. However, in practice, triple probes measurements are subjected to electronic noise, probe alignment variations, and voltage fluctuations. Even under strictly Maxwellian conditions, random measurement uncertainties introduced in the varying currents and voltages have the potential to propagate into significant uncertainty when calculating the electron temperature and number density.

The present work quantifies the effects of measurement uncertainty on plasma properties that were calculated based on triple probe data under Maxwellian conditions. A physics-based forward model is used to simulate a triple probe current collection and create an ideal I-V characteristic curve from the probe measurements. Controlled perturbations are then introduced into the system to mimic uncertainty which would occur in practice. A Monte Carlo analysis is employed to propagate the perturbations through the standard triple probe back calculations, allowing a statistical analysis to be performed on the inferred electron temperature and number density. This framework establishes a bound of accuracy for triple langmuir probe measurements operating under the most ideal conditions and provides an insight into sensitivity to current and voltage uncertainties.

In conventional triple probe analysis, the voltage differences between the probe tips are assumed to be constant throughout the whole experiment. However, in practice, the voltage can vary due to transient effects which occur in plasmas or due to experimental hardware circuitry which may not respond to rapid variations in plasma properties as quickly as needed to gather accurate data. If the plasma develops on a time scale quicker than the circuitry's time responses, there is a major source of error present. This present work aims to quantify the impact that voltage fluctuations and uncertainty has on triple probe measurements.

II. Triple Probe Theory

In Langmuir probes, electrical current is collected through a conducting surface which is inserted into the plasma. Since the plasma is assumed to be Maxwellian, the current that is collected by the biased probe can be expressed as the sum of the ion saturation current and the exponential electron current term. Triple probe theory uses less complex single probe theory as a foundation and results in a closed form solution that establishes a relationship between the collected current and electron temperature through an exponential Boltzmann factor [1,4].

a. Electron and Ion Current Collection

For a plasma in equilibrium, a Maxwellian electron distribution can be assumed and expressed one dimensionally as a function of velocity, v where m_e is the electron mass, k_B is the Boltzmann's constant, and T_e is the electron temperature.

$$f(v) = \sqrt{\frac{m_e}{2\pi k_B T_e}}\, exp\left(-\frac{m_e v^2}{2k_B T_e}\right) \tag{1}$$

The electron flux to a probe biased as voltage V relative to the plasma potential is obtained by integrating over velocities sufficient to overcome the potential barrier. The minimum required velocity is

$$v_b = \sqrt{\frac{-2eV}{m_e}}. \tag{2}$$

The electron current collected by a probe of surface area A can be expressed as

$$I_e(V) = eA_p n_e \sqrt{\frac{k_B T_e}{2\pi m_e}}\, exp\left(-\frac{e(\phi_p - V)}{k_B T_e}\right)\ for\ V < \phi_p \tag{3}$$

where A_p is probe area, T_e, n_e, and e are electron temperature, number density, and charge respectively, and ϕ_p is the plasma potential. For $V > \phi_p$, the exponential barrier vanishes and the electron saturation term can be expressed as

$$I_{e,sat} = eA_p n_e \sqrt{\frac{k_B T_e}{2\pi m_e}}. \tag{4}$$

Under the assumption $T_e >> T_i$, the ion current into the probe can be approximate using the Bohm criterion to

$$I_i = eA_p n_e \sqrt{\frac{k_B T_e}{m_i}}\, exp\left(-\frac{1}{2}\right) \tag{5}$$

where m_i is the ion mass. In standard triple probe theory [2], the ion current is assumed to be independent of probe bias within the operating range of the triple probes. The net current collected is then

$$I_p(V) = I_e(V) - I_i. \tag{6}$$

b. Triple Langmuir Probe Analysis

The triple probe may operate in either voltage mode or current mode. In this present work, current mode is employed (figure 1). In this configuration, three probe tips with a voltage difference are inserted into the plasma. In an ideal case, this voltage difference is fixed; however in actuality, this voltage difference fluctuates.

Within this configuration, the currents satisfy the current continuity constraint

$$I_{p1} + I_{p2} + I_{p3} = 0 \tag{7}$$

where the voltage difference are defined as

$$V_1 - V_2 = V_{d2} \tag{8a}$$

$$V_2 - V_3 = V_{d3} \tag{8b}$$

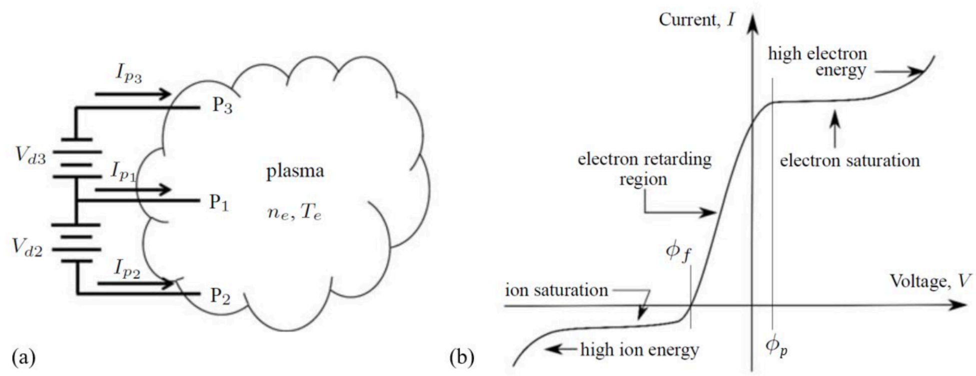


Figure 1: (a) Triple Probe Schematics in Current Mode, (b) Single-Probe Current-Voltage (I-V) Illustration.

Assuming Maxwellian electron behavior, the collected currents can be expressed as

$$I_{p1} = I_e\left(V_1\right) - I_i \tag{9a}$$
$$I_{p2} = I_e\left(V_2\right) - I_i \tag{9b}$$
$$I_{p3} = I_e\left(V_3\right) - I_i \tag{9c}$$

Using the equilibrium equations and voltage relations, the electron temperature can be rearranged into the following current ratio

$$\frac{I_{p1}-I_{p2}}{I_{p1}-I_{p3}} = \frac{exp\left(\frac{eV_1}{k_BT_e}\right)-exp\left(\frac{eV_2}{k_BT_e}\right)}{exp\left(\frac{eV_1}{k_BT_e}\right)-exp\left(\frac{eV_3}{k_BT_e}\right)} = \frac{1-exp\left(-\frac{eV_{d2}}{k_BT_e}\right)}{1-exp\left(-\frac{eV_{d3}}{k_BT_e}\right)} \tag{10}$$

Equation 10 is transcendental in T_e and can be solved numerically. Once the electron temperature is determined, the electron number density can be inferred from the ion saturation current using the following

$$n_e = \frac{I_i}{eA_p}\sqrt{\frac{m_i}{k_BT_e}}\, exp\left(\frac{1}{2}\right) \tag{11}$$

This two-step procedure where T_e is calculated then followed by n_e forms the foundations for standard triple probe analysis and is used throughout the present study.

c. Model Assumptions and Operational Conditions

This analysis was conducted under classical triple langmuir probe assumptions. These assumptions define the operating regime under which the back calculations described above are valid. First, the electron energy distribution is assumed to be Maxwellian. This assumption allows for the exponential form of the electron current (eqn. 3) and the current ratio formula (eqn. 10) to be utilized. Deviations from the Maxwellian assumption, such as multi-temperature distribution and suprathermal electrons, are not considered in this work. Second, the ions are assumed to satisfy the Bohm sheath criterion where electron temperature is much greater than ion temperature. Under this condition, the ion saturation current is treated independent of the probe bias in the voltage range. This assumption allows for cancellation of the ion current term in terms of the current ratio formula, which is used to determine electron temperature.

Thirdly, the probe trips are assumed to be identical to each other in both surface area and collection characteristics. Effects like sheath expansion differences and geometric asymmetry are neglected here. The plasma is also assumed to be spatially uniform across the probe distances. Finally, the applied voltage differences, V_{d2} and V_{d3} are treated as fixed parameters. However, in practice, the voltages may observe some deviations due to noise in the power supply or circuit. The impact of this voltage uncertainty on the inferred plasma properties is investigated in this present work through controlled voltage differences perturbations. Under these assumptions, any deviation between the stated plasma parameters and the inferred parameters can be attributed to uncertainty in the collected currents and applied voltages. This framework works to establish a baseline for future studies which incorporate non-Maxwellian effects to be compared against.

III. Simulation Strategy

The governing equations outlined in section II are implemented numerically to simulate triple Langmuir probe measurements and to quantify the effect of measurement uncertainty on the recovered plasma parameters. A forward model is first employed to generate ideal probe currents for the designated plasma state after which controlled perturbations are introduced into either the measured currents or applied voltage differences. The triple probe inversion procedure is then applied to the recovered electron temperatures and number densities and a Monte Carlo simulation is performed to statistically evaluate the resulting uncertainty distributions.

a. Nominal Plasma Baseline Conditions

All of the simulations ran were performed at a set nominal plasma condition with a set electron temperature and number density. The plasma is also assumed to be Maxwellian and uniform over the probe area for this baseline measurement. Nominal probe voltage differences are also selected and assumed to be fixed. The nominal values show the steady state voltages that would be applied under ideal conditions. The purpose of this study is not to examine multiple plasma states, but to isolate and quantify the effect of uncertainty in the applied voltage.

b. Current Measurement Uncertainty

Uncertainty in the collected probe currents is modeled by introducing independent gaussian perturbations to each probe current:

$$I^{(k)}_{pi} = I_{pi} + \sigma_i N(0, 1) \tag{12}$$

where the uncertainty level represents Gaussian noise with an associated standard deviation, σ_1. Perturbation levels that represent typical experimental noise are considered here. The perturbed currents are substituted into the triple probe current ratio formula (eqn. 10) and then the electron temperature is numerically solved. The electron number density is then calculated using the Bohm ion saturation current expression and this process is repeated for ten thousand Monte Carlo simulations, creating a statistical distribution of the inferred plasma properties. Since current uncertainty enters the inversion through differences and ratios of measured currents, its propagation is expected to remain approximately proportional for small perturbations. The results of this analysis establish a baseline sensitivity under ideal conditions.

Past works [5] have shown that at higher electron temperatures, more negative probe bias is required to fully repel the electrons, violating the assumption that probe three operates strictly in the ion saturation region. While the ion saturation assumption violation at high electron temperatures is noteworthy, the present work focuses only on uncertainty which arises from random measurement variations and the observed behavior demonstrates another source of error entirely - sensitivity of triple probes to model assumptions. The combination of measurement uncertainty and model assumption sensitivity can even further skew the accuracy of inferred plasma properties.

c. Voltage Measurement Uncertainty

Uncertainty in voltage differences is treated separately since current and voltage play fundamentally different roles in the triple probe inversion process. Since the voltage differences occur inside the exponential Boltzmann factor, the uncertainty introduced into the system is nonlinear. The nominal plasma state for the voltage uncertainty analysis is set using an electron temperature of 5 eV, a number density of 10^{18} m^{-3}, and voltage differences V_{d2} and V_{d3} of 6 and 9 volts respectively assuming ideal Maxwellian plasma conditions. Instead of introducing random perturbations, as done in the current analysis, the applied voltages are systematically offset from their nominal values by a fixed percentage (5%, 10%, and 15%) to examine the sensitivity of the inversion process when the voltage differences deviate from their nominal values.

d. Uncertainty Modeling Procedure

For each measurement uncertainty level, systemic uncertainty is introduced to the probe measurements and the inversion process is repeated over a large number of Monte Carlo runs. In each run:

1. Probe voltage differences are adjusted from their nominal values by a fixed offset,
2. Model is evaluated at altered voltages and currents are computed,
3. Gaussian noise is applied to the computed currents,
4. Electron temperature is inferred using standard triple probe inversion while assuming applied voltages remained at their nominal values,
5. Number density is inferred while operating under the assumption that the third probe current is the ion saturation current.

This method allows for a direct comparison between current uncertainty propagation and voltage uncertainty propagation in the inferred plasma properties, while also keeping the same nominal plasma conditions and inversion process. This method also highlights the nonlinear sensitivity of the triple probe inversion process to voltage offsets and can mimic realistic experimental results where the operator is unaware of the voltage deviations.

IV. Results and Discussion

This section briefly reviews the results from a previous paper which examined current uncertainty propagation and highlighted the results of the measured current uncertainty for the selected test case. The selected test case of electron temperature of 5 eV and a number density of 10^{18} m^{-3} is then examined further in the voltage uncertainty analysis.

a. Current Uncertainty Propagation

Effects of random measurement uncertainty on inferred plasma properties, specifically in current measurements, have previously been investigated [5]. In that work, independent gaussian perturbations were applied to the probe currents and the electron temperatures and number densities were recovered using the standard triple probes inversion. The uncertainties (1%, 5%, and 10%) were applied to different combinations of electron temperatures and number densities. Prior to running the Monte Carlo, a set of baseline was established - one of the electron temperature and number density combinations were 5eV and $10^{18}\,m^{-3}$ respectively (table 1). That case is chosen to be further evaluated in the voltage uncertainty analysis.

Table 1. Forward Model Verification Results for Test Case

Input		Unperturbed Currents			Recovered		Error [%]		Sum [mA]
$T_e\ [eV]$	$n_e\ [m^{-3}]$	$I_{p1}\ [mA]$	$I_{p2}\ [mA]$	$I_{p3}\ [mA]$	$T_e\ [eV]$	$n_e\ [m^{-3}]$	T_e	n_e	$I_{p1} + I_{p2} + I_{p3}$
5	10^{18}	3.34	-1.21	-2.11	5.03	10^{18}	0.632	0.257	0.0222

The analysis demonstrated that uncertainty scales linearly with respect to the applied current noise. The distributions of the inferred electron temperature and number densities were shown to be approximately gaussian demonstrating the normality of the Monte Carlo simulation. This established that under ideal Maxwellian conditions, current noise alone does not introduce significant nonlinear effects in the triple probe inversion process. These results can lay a foundation for further uncertainty analysis such as voltage uncertainty.

b. Voltage Uncertainty Analysis

Unlike current uncertainty, which propagates approximately linearly in the inversion process, voltage uncertainty enters the system through the exponential Boltzmann factor. As a result, even small deviations in the applied voltage differences can introduce nonlinear effects into the inversion process. This will result in the inferred electron temperature and number density to be significantly more sensitive to voltage deviations than to current noise.

Just like the current uncertainty analysis, a baseline case must first be established before the introduction of measurement uncertainty. Using the nominal Maxwellian plasma state of electron temperature of 5 eV and number density of m^{18} m^{-3}, with voltage differences of V_{d2} and V_{d3} of 6 and 9 volts respectively, a reference data set for the voltage uncertainty analysis can be generated (table 2) by applying systemic voltage off sets of $\pm$5%, $\pm$10%, and $\pm$15% to the applied nominal voltage differences. For each voltage offset, the forward model is utilized to calculate the corresponding probe currents. The currents are used in the inversion process while assuming the voltages remain at their nominal values.

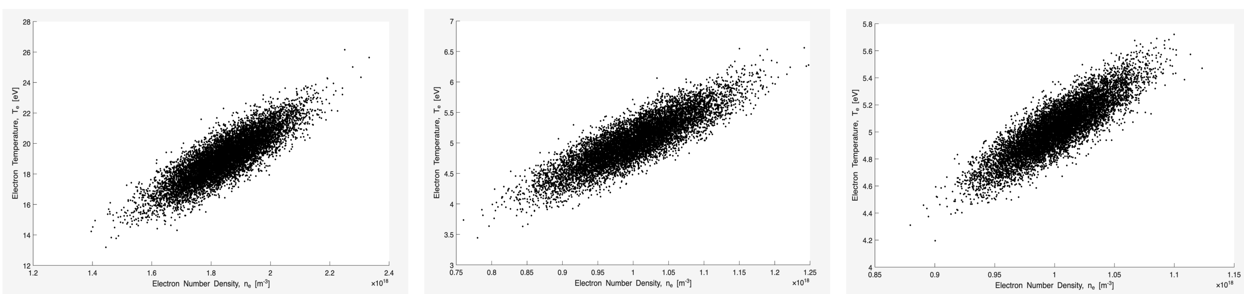
Table 2. Baseline Data Set of $T_e = 5\ eV$ and $n_e = 10^{18}\ [m^{-3}]$

Input		Current Info				Recovered		% Error	
Vd2	Vd3	Ip1	Ip2	Ip3	I sum	Te	ne	Te	ne
5.1	7.65	0.00286	-0.00101	-0.00187	-2.18×10^{-05}	19.104	1.85×10^{18}	282.01	84.75476434
4.5	8.1	0.00301	-0.00107	-0.00195	-1.29×10^{-06}	8.690	1.23×10^{18}	73.79	23.41908901
5.7	8.55	0.00314	-0.00116	-0.00204	-5.86×10^{-05}	6.151	1.06×10^{18}	23.02	6.448893026
6	9	0.00334	-0.00121	-0.00211	2.22×10^{-05}	5.0316	1.00×10^{18}	0.63	0.257310689
6.3	9.45	0.00347	-0.00130	-0.00218	-5.92×10^{-06}	4.356	9.78×10^{17}	12.88	2.174310327
6.6	9.9	0.00359	-0.00136	-0.00224	-1.13×10^{-05}	3.973	9.77×10^{17}	20.54	2.317542443
6.9	10.35	0.00376	-0.00144	-0.00231	2.29×10^{-05}	3.637	9.88×10^{17}	27.26	1.210877263

As expected, higher voltage deviations produce high error in the recovered electron temperatures and number densities. Even minor voltage offsets result in a large bias, especially in the electron temperature. In the most extreme case, the recovered temperature has a percentage difference more than 250%, highlighting the nonlinear amplification effect.

Once the baseline data set is established, a Monte Carlo analysis is performed for each voltage offset. In each run, the forward model evaluates the currents at the systematically shifted voltages, Gaussian perturbations are then applied to the currents to mimic measurement noise, and the electron temperatures and number densities are inferred assuming the nominal voltage differences. The recovered electron temperatures and number densities are then statistically analyzed - examining the mean, median, standard deviation, percentage uncertainty, and skewness (table 3). The results show that the voltage uncertainty is nonlinearly amplified in the recovered number densities. Interestingly, at the largest voltage deviation level, the inferred number density shifts slightly back towards the true value.

Scatter plots of the inferred electron temperatures and number densities are created for three representative voltage differences: the lowest offset of V_{d2} = 5.1 V and V_{d3} = 7.65 V, the nominal offset of V_{d3} = 6 V and V_{d3} = 9 V, and the highest offset of V_{d2}= 6.9 V and V_{d3} = 10.35 V. The lowest voltage difference case shows the widest range of scatter far from the true plasma state, highlighting the nonlinear amplification of voltage uncertainty in the inversion process. The nominal case shows a tighter range of scattering around the true values, while the high voltage difference case shows an intermediate scatter. These plots can help visualize the sensitivity of the inferred plasma parameters to systematic voltage deviations.



(a) **(b)** **(c)**

Figure 2: Representative scatter plots of all Monte Carlo data points for the case T_e= 5 eV and $n_e = 10^{18}$ m^{-3} at 5% uncertainty for voltage differences of (a) V_{d2} = 5.1 V and V_{d3} = 7.65 V, (b) V_{d3} = 6 V and V_{d3} = 9 V, and (c) V_{d2}= 6.9 V and V_{d3} = 10.35 V.

Histograms of the same three voltage difference cases provide further insight into the deviations of electron temperature and number density when uncertainty is applied to the voltages. The off-nominal cases, in particular the lowest voltage difference, shows substantial deviations from the true values, highlighting the impact voltage offsets can have on the returned electron temperatures and number densities. The nominal and high voltage difference histograms remain closer to the baseline, demonstrating reduced sensitivity, but still present deviations. All together, the scatter plots and histograms work to highlight the nonlinear and amplified effect that voltage uncertainty can have on triple probe measurements.

The data and figures previously shown demonstrate how voltage uncertainty has a strong nonlinear effect on the recovered plasma parameters. Specifically, how small deviations in the applied probe voltages lead to substantial over or underestimation of the electron temperature, with errors exceeding 250% in extreme cases, while electron density showed smaller but still significant deviations. The scatter plots show that the distribution of inferred values is highly dependent on the magnitude and direction of the voltage offsets. Lower voltage differences produce a wider scatter and biased results, nominal voltages yield a tighter cluster of data near the true plasma parameter, and higher voltages show intermediate scatter with a partial recovery of number density. The histograms highlighted the distributions for off-nominal voltages and showed the non-Gaussian nature of the errors. All together, these results demonstrate how voltage deviations introduce nonlinear amplification effects not seen in current noise alone, highlighting the need for precise voltage control in triple probe measurements to avoid systematic errors.

Table 3. Monte Carlo Output Summary of Voltage Uncertainty

Voltage Uncertainty	Mean Te [eV]	Median Te [eV]	σ_{Te} [eV]	σ_{Te} /Mean Te [%]	Te Skewness	Mean ne [m^-3]	Median ne [m^-3]	σ_{ne} [m^-3]	σ_{ne} / Mean ne [%]	ne skewness
Input Vd2 = 5.1, Vd3 = 7.65										
0.01	19.1051	19.1041	0.152	0.80	0.0289	1.847×10^{18}	1.847×10^{18}	1.169×10^{16}	0.63	0.0159
0.05	19.1122	19.1081	0.7458	3.90	0.0202	1.848×10^{18}	1.848×10^{18}	5.726×10^{16}	3.10	0.0126
0.10	19.1084	19.0853	1.5033	7.87	0.0324	1.848×10^{18}	1.848×10^{18}	1.144×10^{17}	6.19	0.0274
Input Vd2 = 5.4, Vd3 = 8.1										
0.01	8.6901	8.6903	0.0678	0.78	-0.029	1.234×10^{18}	1.234×10^{18}	7.687×10^{15}	0.62	-0.0154
0.05	8.6923	8.691	0.3373	3.88	0.0321	1.235×10^{18}	1.234×10^{18}	3.769×10^{16}	3.05	0.0277
0.10	8.6972	8.6922	0.6882	7.91	0.0163	1.234×10^{18}	1.234×10^{18}	7.696×10^{16}	6.24	0.0268
Input Vd2 = 5.7, Vd3 = 8.55										
0.01	6.1507	6.1505	0.0484	0.79	0.0009	1.064×10^{18}	1.064×10^{18}	6.648×10^{15}	0.62	0.0017
0.05	6.1528	6.1544	0.2399	3.90	-0.028	1.065×10^{18}	1.065×10^{18}	3.296×10^{16}	3.10	-0.0077
0.10	6.15	6.1454	0.4783	0.48	-0.006	1.064×10^{18}	1.064×10^{18}	6.591×10^{16}	6.19	-88
Input Vd2 = 6, Vd3 = 9										
0.01	5.0316	5.0314	0.0391	0.78	-0.0062	1.003×10^{18}	1.003×10^{18}	6.16×10^{16}	0.61	-0.0149
0.05	5.0316	5.0319	0.1977	3.93	0.0073	1.003×10^{18}	1.003×10^{18}	3.137×10^{16}	3.13	0.0003
0.10	5.0304	5.0308	0.3913	7.78	-0.0161	1.003×10^{18}	1.003×10^{18}	6.221×10^{16}	6.20	-0.0098
Input Vd2 = 6.6, Vd3 = 9.9										
0.01	3.9735	3.9735	0.0306	0.77	0.0181	9.769×10^{17}	9.77×10^{17}	6.006×10^{16}	0.61	-0.208
0.05	3.9706	3.9714	0.1561	3.93	-0.0132	9.763×10^{17}	9.763×10^{17}	3.046×10^{16}	3.12	0.0226
0.10	3.9735	3.97	0.3099	7.80	0.0286	9.768×10^{17}	9.767×10^{17}	6.078×10^{16}	6.22	-0.0044
Input Vd2 = 6.9, Vd3 = 10.35										
0.01	3.6367	3.6364	0.0283	0.78	0.0174	9.879×10^{17}	9.879×10^{17}	6.09×10^{15}	0.62	0.0058
0.05	3.6372	3.6371	0.1396	3.84	0.0164	9.88×10^{17}	9.879×10^{17}	3.021×10^{16}	3.06	0.0151
0.10	3.6345	3.6361	0.2787	7.67	-0.0189	9.877×10^{17}	9.879×10^{17}	6.021×10^{16}	6.10	-0.0213

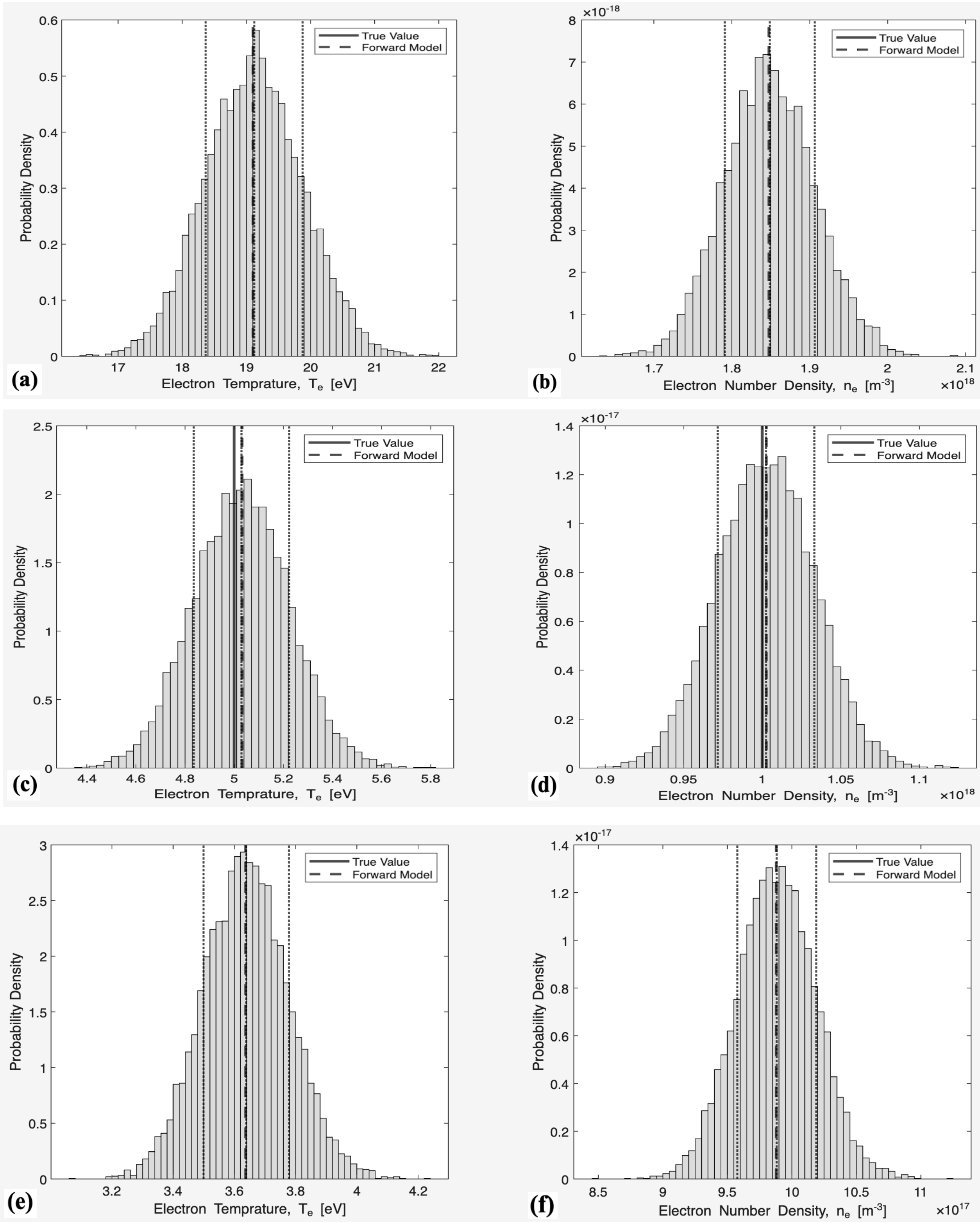


Figure 3: Representative histograms of number density and electron temperature for the case T_e = 5 eV and $n_e = 10^{18}$ m^{-3} at 5% uncertainty for voltage differences of a) and b) at V_{d2} = 5.1 V and V_{d3} = 7.65 V, c) and d) at V_{d3} = 6 V and V_{d3} = 9 V, and e) and f) at V_{d2}= 6.9 V and V_{d3} = 10.35 V. Shown on the histograms are the initial inputs (true values) and the mean and standard deviations of the data sets.

V. Conclusion

This work quantifies the effects of measurement uncertainty on triple Langmuir probes under ideal Maxwellian plasma conditions by using a forward model with a Monte Carlo simulation. The propagation of both current and voltage uncertainty into the inferred electron temperature and number density are examined. As expected, current uncertainty propagates approximately linearly through the inversion process, yielding a nearly Gaussian distribution in the recovered electron temperatures and number densities.

In contrast, voltage uncertainty appears in the inversion process nonlinearly in the exponential Boltzmann factor resulting in deviations of the inferred electron temperature and number density. Scatter plots and histograms illustrate off-nominal voltage differences and show how they can cause the inferred plasma parameters to be far from the true values, highlighting the sensitivity of triple probes to voltage offsets. It is noteworthy that at the largest voltage deviation, the recovered number density actually shifts slightly back toward the true value instead of continuing to move further away, demonstrating the complex, nonlinear behavior present in the inversion process.

Overall, this work establishes a quantitative baseline for interpreting triple probe measurements under ideal Maxwellian conditions. It also examines experimental uncertainty and provides insight into the impact of current perturbation versus voltage perturbations, which is important to understand for plasma diagnostics when dealing with transient plasmas. Future work may involve developing a master simulation that simultaneously evaluates voltage and current uncertainties for transient plasmas.

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