Prediction of Fin Flutter via Modal Analysis

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Aeroelastic analysis on rocket fins is difficult to perform at the amateur and collegiate levels due to access issues with capable software tools. Providing MATLAB code for analytical solutions to flutter in tandem with modal FEA and physical testing removes accessibility barriers while preserving reasonable accuracy. Multiple analytical methods are presented with comparison between them. Work is ongoing on validating modal FEA with vibration testing.

I. Nomenclature

a	=	speed of sound (ft/s)
a_h	=	non-dimensional location of elastic axis
b	=	semi-chord (in)
c	=	chord length (in)
G_E	=	shear modulus (psi)
g	=	damping factor
g_α, g_h	=	torsional and bending damping coefficients, respectively
k	=	reduced frequency
M	=	Mach number
m	=	mass per unit span
p_0	=	standard air pressure at sea level (psi)
p	=	air pressure (psi)
r_α	=	reduced radius of gyration
t	=	fin thickness (in)
V	=	velocity (ft/s)
V_d	=	divergence velocity (ft/s)
V_f	=	flutter velocity (ft/s)
x_α	=	semi-chord normalized location of center of gravity behind elastic axis
μ	=	mass ratio, $\mu = \frac{m}{\rho b^2}$
ρ	=	air density (lbm/in^2)
γ	=	ratio of specific heats
ω, ω_f	=	frequency, flutter frequency (rad/s)
ω_α, ω_h	=	natural torsion and bending frequencies, respectively (rad/s)
ϵ	=	chord-normalized distance of center of gravity behind quarter-chord
λ	=	chord taper ratio
AR	=	aspect ratio

II. Introduction

Aeroelasticity is the study of the interaction between changing aerodynamic forces and non-rigid structures, such as wings or rocket fins. Aeroelastic phenomena can arise for many different reasons, such as the pressure difference between the interior and exterior of a hollow structure, maneuvers taken during flight, and sudden changes in air speed/direction due to wind. Aeroelastic analysis is difficult due to the highly nonlinear nature of aerodynamic forces and the interdependent nature of the aerodynamic-structural interaction.

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Fin flutter is an aeroelastic phenomena observed on high-power model rockets, like those built by the Mississippi State University Space Cowboys, where at least one of the fins on a rocket begins to oscillate (“flutter”) in flight. The best outcome for this is that the rocket only experiences a loss in flight performance, however, fin flutter typically causes vehicles to break apart in flight, especially while supersonic. The performance loss and safety risk are both undesirable, leading to competitions like the International Rocket Engineering Competition (IREC) to require teams to perform some form of flutter analysis.

Software such as ZAERO, ANSYS, and NASTRAN are capable of aeroelastic analysis, but require some combination of expensive licenses, large amounts of computing power, advanced understanding of the conditions at play, and advanced technical knowledge to produce any, much less usable or accurate, results. These hurdles pose an accessibility challenge to the majority of the hobby and college rocketry community, which results in engineers turning to analytical software such as AeroFinSim (no longer distributed) or simple methods based on [1] and [2] calculated in spreadsheets. Such methods, while accessible, produce inaccurate results that lead to either over-designed, heavy vehicles or in-flight failures below expected limits.

By leveraging the computing and testing resources available at Mississippi State University, this project aims to create an accurate but accessible method by which flutter can be predicted for common rocket fin geometries and materials.

III. Analytical Approach

A MATLAB script was written to collect various analytical methods of calculating flutter velocity, U_f, for easy comparison. Included are the TN4197 (from [1]), Bennett (from [2]), $1/k$ (from [3, 4] with a formulation from [5]), U vs. g(from [6, 7] with a formulation from [5]), and Simmons Supersonic (from [8]) methods.

A. TN4197 Method

The method from [1] is presented as a simplified calculation for predicting flutter in a fin based upon the formula presented in [4]. Several assumptions and approximations are made; It is assumed that the fin has a high aspect ratio, a low ratio between the bending and torsion frequencies, a constant thickness to chord ratio that allows the torsional frequency to be easily approximated, and assumes that the center of gravity is located near the half-chord location. The high aspect ratio, low frequency ratio, and constant thickness to chord ratio are generally untrue in modern rocketry applications. Despite this, the method is widely applied in amateur rocketry for its simplicity, shown in (1):

$$(\frac{V_f}{a})^2 = \frac{G_E}{\frac{39.3 AR^3}{(\frac{t}{c})^3 (AR+2)} (\frac{\lambda+1}{2})(\frac{p}{p_0})} \tag{1}$$

The method is further complicated by the use of advanced composite materials, such as fiberglass and carbon fiber, which display anisotropic behavior. This, however, is outside the scope of this paper.

B. Bennett Method

The method from [2] attempts to correct the shortcomings of [1], primarily by removing the assumption of the location of the fin center of gravity, and providing an approximation for its location. The method however does not address the constant thickness ratio, low frequency ratio, or high aspect ratio assumptions from [1]. Bennett's publication includes (2) and demonstrates a 39.6% difference between the center of gravity parameter calculated in his paper and the original presented in [1] with a worked example.

$$V_f = a \times \sqrt{\frac{G_E}{\frac{DN AR^3}{(\frac{t}{c})^3 (AR+2)} (\frac{\lambda+1}{2})(\frac{p}{p_0})}} \tag{2}$$

with*

$$DN = \frac{24 \epsilon \gamma p_0}{\pi} \tag{3}$$

*Bennett uses κ in place of γ for the ratio of specific heats in his paper. γ is used here for consistency with general convention.

C. $1/k$ **Method**

The $1/k$ method is originally presented in [3, 4] and referenced in [1, 6, 7]. This method is also one of the two provided in AeroFinSim. This method revolves around *Theodorsen's Circulation function,* $C(k)$, which relates the forcing and response of the fin. The original approach from 1940 is performed graphically, requiring the plotting of the real and imaginary portions of the function, $\sqrt{X}$, against the reciprocal of the reduced frequency, k, and finding the intercept point of the two curves. This did not require lengthy iteration or extensive recalculation of the Bessel functions necessary to $C(k)$, however this is difficult to implement in code; While a direct solution is extremely difficult, MATLAB makes the calculation of $C(k)$ trivial and thus allows a solution to be found by sweeping over a range of k values. In code, this is done by creating a vector of $C(k)$ values and applying them to

$$\Delta_R = (1 - g_h g_\alpha)\mu^2 r_\alpha^2 \frac{\omega_h^2}{\omega_\alpha^2} X^2 + (\mu \frac{\omega_h^2}{\omega_\alpha^2}(E_R - g_h E_I) + \mu r_\alpha^2 (A_R - g_\alpha A_I))X + A_R E_R - B_R D_R - A_I E_I + B_I D_I \quad (4)$$

$$\Delta_I = (g_h + g_\alpha)\mu^2 r_\alpha^2 \frac{\omega_h^2}{\omega_\alpha^2} X^2 + (\mu \frac{\omega_h^2}{\omega_\alpha^2}(g_h E_R + E_I) + \mu r_\alpha^2 (A_I + g_\alpha A_R))X + A_I E_R - B_R D_I + A_R E_I - B_I D_R \quad (5)$$

calculating the roots of (4) and (5) as X_R and X_I respectively, taking the square roots of X_n respectively, then finding the minimum ratio between the square roots to determine the flutter point, and calculating V_f as

$$V_f = \frac{\omega_\alpha b}{k\sqrt{X}} \quad (6)$$

with flutter frequency ω_f

$$\omega_f = \frac{\omega_\alpha}{\sqrt{X}} \quad (7)$$

$A_R, A_I, B_R, B_I, D_R, D_I, E_R$, and E_I (and their components) can be found in [6]. Notably, this method contains structural damping terms g_h and g_α unlike the previously discussed methods. These terms account for energy loss within the fins that is unaddressed in the TN4197 and Bennett methods, providing more realistic results.

D. U **vs.** g **Method**

The U vs. g method is presented in [6] and [7] as an alternative to the $1/k$ method for flutter prediction that utilizes $C(k)$, and is the other method provided in AeroFinSim. This method defines the flutter condition where the value of g, an imaginary damping coefficient, crosses from the underside of the x-axis. The original form presented required lengthy iteration and was more difficult to solve directly than the $1/k$ method. [5] provides a matrix formulation solved as an eigenvalue problem that is trivial to compute in MATLAB:

$$\Omega \begin{bmatrix} \bar{h}/b \\ \bar{\alpha} \end{bmatrix} = \begin{bmatrix} \frac{\omega_\alpha^2}{\omega_h^2}(1 + \frac{L_h}{\mu}) & \frac{\omega_\alpha^2}{\omega_h^2}(x_\alpha + \frac{1}{\mu}(L_\alpha - (\frac{1}{2} + a_h)L_h)) \\ \frac{x_\alpha}{r_\alpha^2} + \frac{1}{\mu r_\alpha^2}(\frac{1}{2} - (\frac{1}{2} + a_h)L_h) & 1 + \frac{1}{\mu r_\alpha^2}(M_\alpha - (L_\alpha + \frac{1}{2})(\frac{1}{2} + a_h) + L_h(\frac{1}{2} + a_h)^2) \end{bmatrix} \begin{bmatrix} \bar{h}/b \\ \bar{\alpha} \end{bmatrix} \quad (8)$$

where Ω is the complex eigenvalue defined by

$$\Omega = \Omega_R + i\Omega_I = (\frac{\omega_\alpha}{\omega})^2 \quad (9)$$

and L_h, L_α, and M_α are nondimensional lift and moment terms defined by

$$L_h = 1 - i\frac{2C(k)}{k} \quad (10)$$

$$L_\alpha = \frac{1}{2} - i\frac{1 + 2C(k)}{k} - \frac{2C(k)}{k^2} \quad (11)$$

and

$$M_\alpha = \frac{3}{8} - \frac{i}{k} \tag{12}$$

In code, this is implemented similarly to the $1/k$ method with a sweep over k to calculate $C(k)$, and then solving for the two eigenvalues of (8) for each value of k. Note that Ω can be represented as

$$\Omega = \Omega_R + i\Omega_I = (\frac{\omega_\alpha}{\omega})^2(1 + i\frac{\Omega_I}{\Omega_R}) = (\frac{\omega_\alpha}{\omega})^2(1 + ig) \tag{13}$$

where g bears similarity to a damping coefficient. The frequency, ω, at any given eigenvalue is defined by

$$\omega = \frac{\omega_\alpha}{\sqrt{\Omega_R}} \tag{14}$$

which provides

$$V = \frac{\omega b}{k} \tag{15}$$

Where the flutter velocity, V_f for each eigenvalue is defined where g is zero, with the lower value of V_f denoted the flutter velocity and the higher value denoted the divergence velocity, V_d. This is visualized by plotting g against V, with the respective x-axis crossings defining the two points. This formulation is identical to the $1/k$ method when $g_h, g_\alpha = 0$ however unlike the $1/k$ method, it provides V_d, providing information on the aeroelastic static instability case; Informally, V_d is the "do not exceed" velocity.

E. Simmons Supersonic Flutter Equation

The Simmons supersonic flutter equation is a simplification of the John Hopkins supersonic flutter equation present in [9]. Much like the $1/k$ and U vs. g methods, it is originally derived from the aeroelastic equations of motion, however it is formulated for supersonic flight, rather than sub- or transsonic flight. It is defined by

$$V_f = \omega_\alpha b \sqrt{\frac{\mu r_\alpha^2 \sqrt{M^2 - 1}}{x_\alpha b} * \frac{(1 - \frac{\omega_h^2}{\omega_\alpha^2})^2 + 4(\frac{x_\alpha}{r_\alpha})^2(\frac{\omega_h^2}{\omega_\alpha^2})}{2(1 + \frac{\omega_h^2}{\omega_\alpha^2})}} \tag{16}$$

with similar parameters to the aforementioned methods, however with the addition of Mach Number. The inclusion of Mach Number makes it attractive for rocketry applications, as the majority of flights of interest are supersonic. However, this poses new implementation challenges, as said flights change both Mach Number and air density rapidly over their course.

IV. Modal Analysis Approach

Flight testing amateur rockets for flutter is difficult, costly, and risky; Instrumentation will have to be custom-made for any vehicle, and will require special care in integration, individual flights can cost hundreds to thousands of dollars in consumables alone and require special waivers from the Federal Aviation Administration and large areas of land to operate safely and legally, and flights risk damaging or losing the vehicle or, though unlikely, injuring a participant in the event of a mishap such as fin shear due to flutter. This makes alternative methods for validation attractive.

With access to shaker systems, assessing flutter velocity by collecting ω_α and ω_h via ground testing of fins and using them in the analytical equations presented above is attractive. For this project, the modal analysis approach involves:

1) Define fin geometry of interest
2) Develop a Computer-Aided Design (CAD) model of a test article fin for use in the shaker system
3) Evaluate the modes and mode shapes of the test article in Finite-Element Analysis (FEA) software to determine modes of interest and prove fitness for testing
4) Use the shaker system to validate FEA models and characterize modes/mode shapes of interest
5) Implement characterized modes into analytical equations to calculate flutter velocity

With sufficient tests, a relationship could be identified between fin geometry and material and the torsion and bending modes of interest that would prove useful for design at hobby scales or as an initial "best guess" in more professional applications without requiring FEA.

A. FEA Modeling Results

Five fin geometries were selected to represent "typical" fins encountered in amateur and collegiate rocketry. Two fins, the "N5800" and "Doppler" fins are close approximations (due to material availability) of actual fins present on existing amateur-scale vehicles, while the other three are "trapezoidal" geometries that were deemed as realistic cases suitable for testing. After the base geometry was selected for each fin, a flange measuring 4 inches by 4 inches with two fastener through holes aligned vertically in the center spaced 2 inches apart was added to each fin, centered at the root mid-chord, to allow them to be mounted to the shaker system at Mississippi State University in a way that emulates traditional "through the wall" fin construction techniques†. Fin geometry is fully defined by the parameters shown in the figure below:

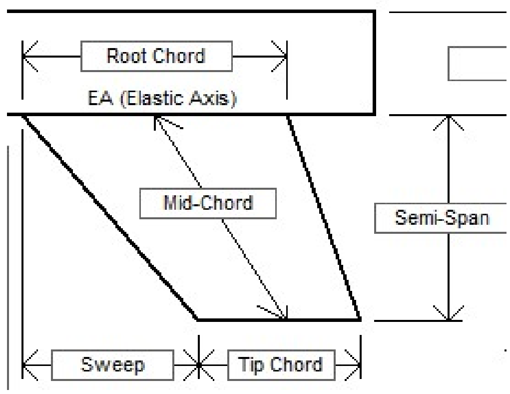


Fig. 1 Screenshot from AeroFinSim showing fin geometry parameters

Where the mid-chord length is a parameter that can be derived from the others. The CAD models were imported into ANSYS Mechanical, constrained with the "fixed support" condition on the faces clamped by the shaker system, and meshed with elements sized at 1/2 the fin thickness along the edges and 0.125 inches on all other faces. The first eight modes were found using the "Modal" analysis system, and mode shape results were generated for all discovered modes. The Torsion and bending modes were selected by viewing the mode shapes, and identifying the lowest frequencies at which pure rotation and pure bending about the elastic axis occurred, respectively.

The N5800 fin is included as an example file with installation of AeroFinSim and was originally created for a high-performance project by the software author John Cipolla. The fin is defined by a 12 inch root chord, 2.25 inch tip chord, 4 inch semi-span, and 9.75 inch sweep length. The original design has a 0.195 inch thickness, but due to material availability a thickness of 0.125 inches was used instead. For testing, the fin is fabricated out of 3003-H14 aluminum alloy. The FEA mesh is shown in Fig. 2

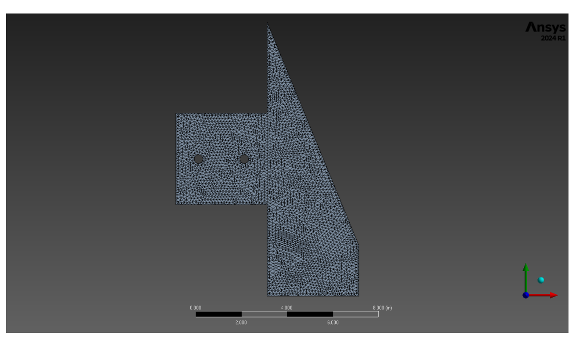
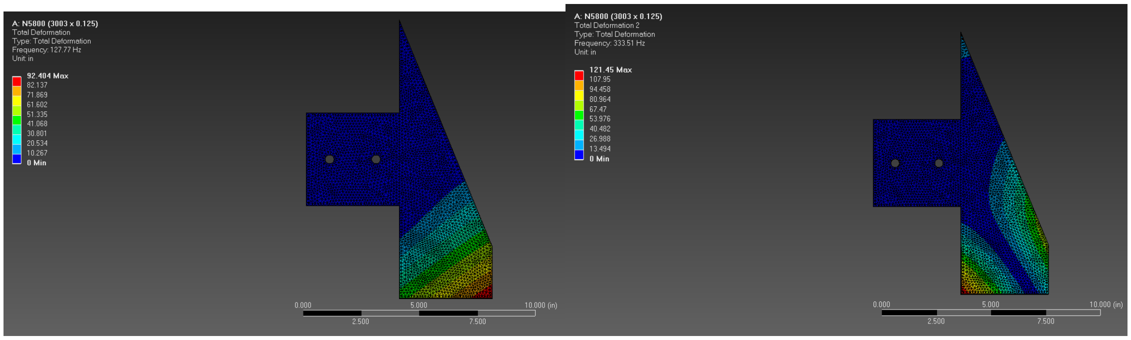


Fig. 2 ANSYS Mechanical Mesh of the N5800 Fin

The analysis was performed and provided a bending mode at 127.77 Hz (ω_h =802.80 rad/s) and a torsion mode at 333.51 Hz (ω_α = 2095.51 rad/s) shown in Fig. 3

†This flange position was also selected due to time constraints. Further investigation into the "best" way to construct test articles is a subject best handled by its own research project.



(a) Bending Mode (b) Torsion Mode

Fig. 3 N5800 Fin bending and torsion modes

The Doppler fin geometry was provided by Everest Sweet for an amateur vehicle called *Doppler Shift*. The fin is defined by a 7 inch root chord, 2 inch tip chord, 3.75 inch semi-span, 7 inch sweep length, and a thickness of 0.250 inches. For testing, the fin is fabricated out of 3003-H14 aluminum alloy. The FEA mesh is shown in Fig. 4

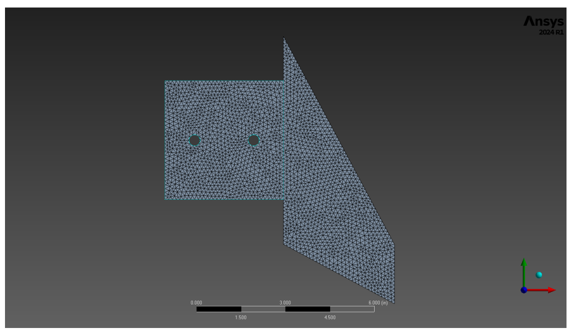
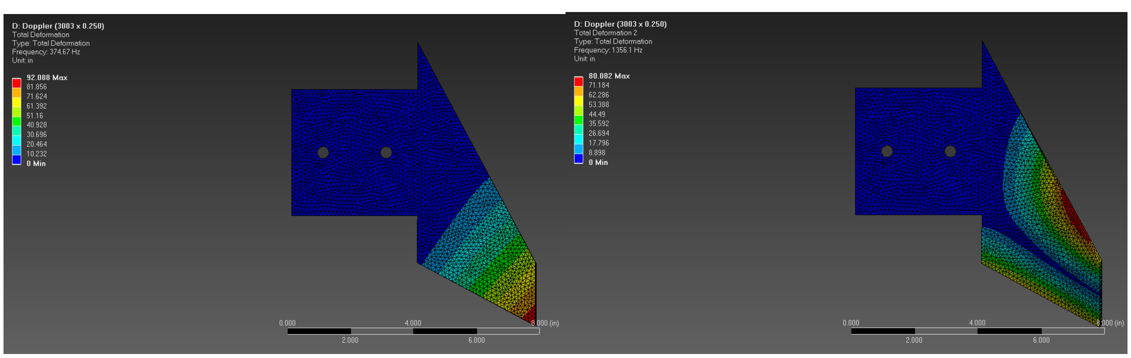


Fig. 4 ANSYS Mechanical Mesh of the Doppler Fin

The analysis was performed and provided a bending mode at 374.67 Hz (ω_h = 2354.12 rad/s) and a torsion mode at 1356.1 Hz (ω_α = 8520.63 rad/s) shown in Fig. 5



(a) Bending Mode (b) Torsion Mode

Fig. 5 Doppler Fin bending and torsion modes

"Fin 1" is a fin geometry determined for the project. The fin is defined by a 10 inch root chord, 2 inch tip chord, 4 inch semi-span, and 8 inch sweep length. The design has a 0.032 inch thickness. For testing, the fin is fabricated out of 2024-T3 aluminum alloy. The FEA mesh is shown in Fig. 6

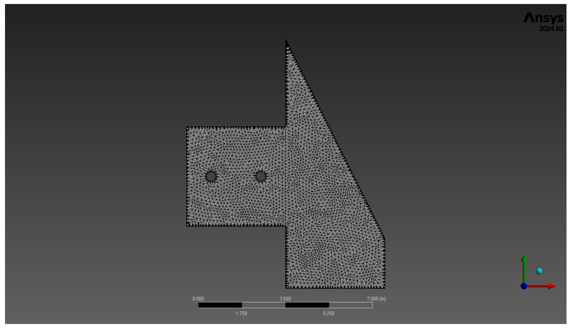
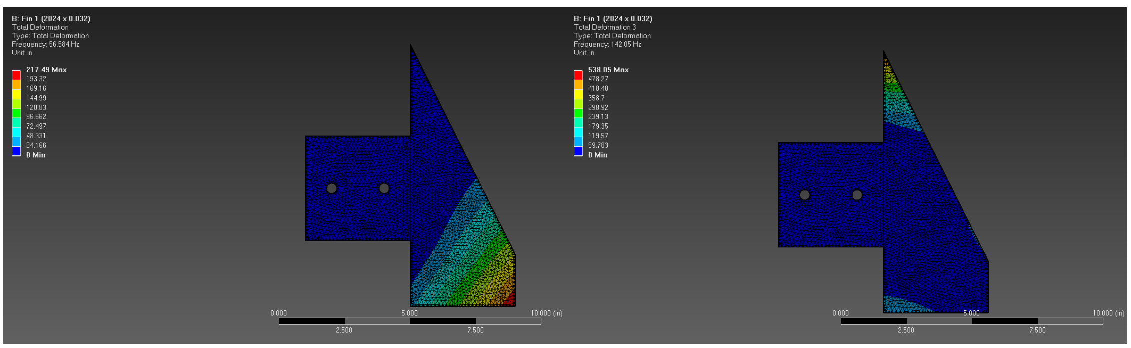


Fig. 6 ANSYS Mechanical Mesh of Fin 1

The analysis was performed and provided a bending mode at 56.584 Hz (ω_h = 355.53 rad/s) and a torsion mode at 142.05 Hz (ω_α = 446.26 rad/s) shown in Fig. 7



(a) Bending Mode (b) Torsion Mode

Fig. 7 Fin 1 bending and torsion modes

"Fin 2" is a fin geometry determined for the project. The fin is defined by a 8.5 inch root chord, 2 inch tip chord, 3 inch semi-span, and 7.5 inch sweep length. The design has a 0.040 inch thickness. For testing, the fin is fabricated out of 2024-T3 aluminum alloy. The FEA mesh is shown in Fig. 8

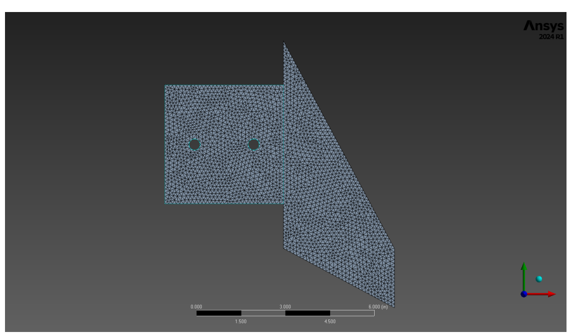
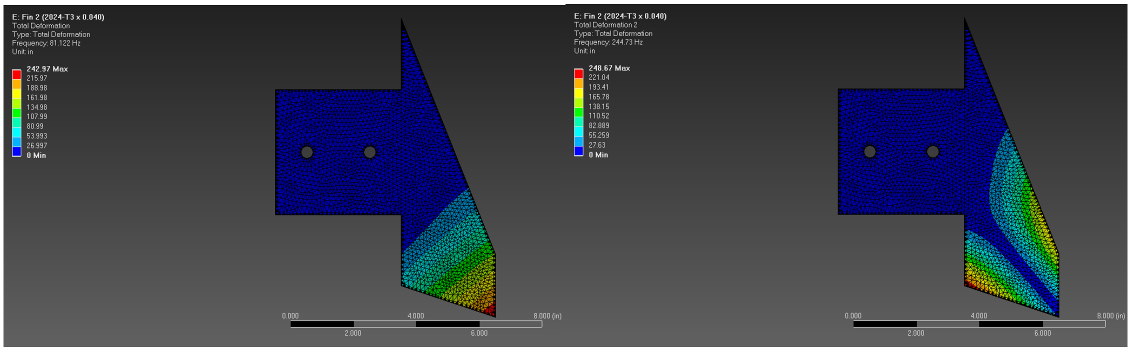


Fig. 8 ANSYS Mechanical Mesh of Fin 2

The analysis was performed and provided a bending mode at 81.12 Hz (ω_h = 509.70 rad/s) and a torsion mode at 244.73 Hz (ω_α = 1537.68 rad/s) shown in Fig. 9



(a) Bending Mode (b) Torsion Mode

Fig. 9 Fin 2 bending and torsion modes

"Fin 3" is a fin geometry determined for the project. The fin is defined by a 6 inch root chord, 2 inch tip chord, 3.5 inch semi-span, and 4 inch sweep length. The design has a 0.125 inch thickness. For testing, the fin is fabricated out of 3003-H14 aluminum alloy. The FEA mesh is shown in Fig. 10

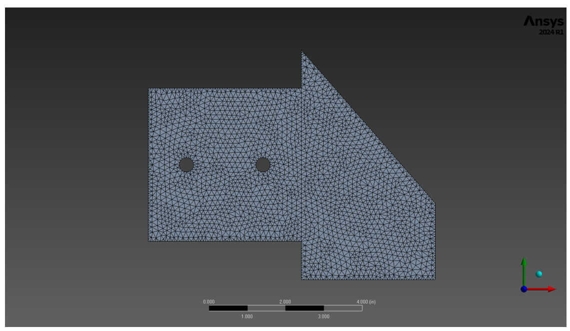
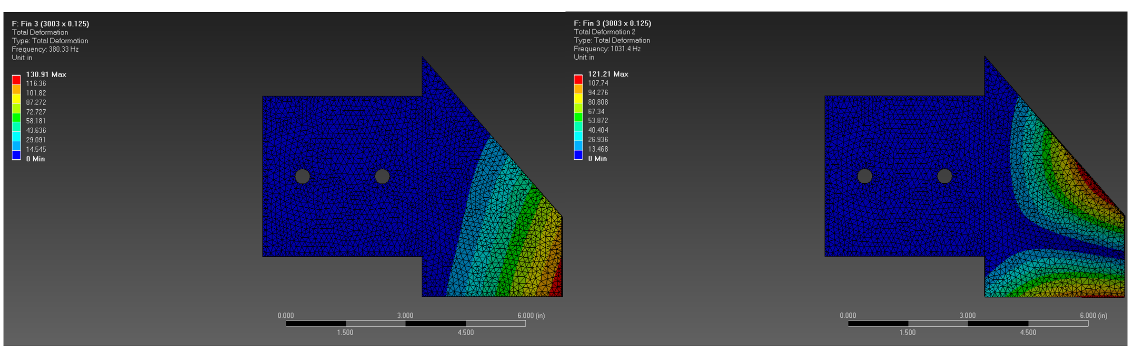


Fig. 10 ANSYS Mechanical Mesh of Fin 3

The analysis was performed and provided a bending mode at 380.33 Hz (ω_h = 2389.68 rad/s) and a torsion mode at 1031.4 Hz (ω_α = 6480.48 rad/s) shown in Fig. 11



(a) Bending Mode (b) Torsion Mode

Fig. 11 Fin 3 bending and torsion modes

B. Instrumentation and Test Procedure

Vibration testing will be conducted using the shaker table in the Fatigue and Fracture Testing Laboratory in Walker Building at Mississippi State University. Test articles will be mounted via the root chord flange to the shaker table, with the root chord oriented parallel to the floor and the fin extending vertically upward, spanning from the mounting surface toward the ceiling. The shaker will excite the fin laterally, driving side-to-side motion perpendicular to the fin face. Response measurements will be collected using ADXL354 accelerometers. Accelerometer placement will follow the guidance of Avitabile [10], with care taken to avoid nodal points identified in the FEA mode shapes to ensure all modes of interest are captured in the frequency response functions (FRFs). FRFs will be processed to extract natural frequencies, damping ratios, and mode shapes, which will then be compared against FEA predictions and used as inputs to the analytical flutter velocity methods described in section III. Test articles have been fabricated and vibration testing is currently ongoing.

V. Conclusion

Work is ongoing on the project. Vibration testing of the fins is planned to be completed before the end of the semester. The collection and comparison of multiple analytical methods for predicting flutter velocity has improved understanding of the applications and shortcoming of each method, and highlighted important areas for further investigation. Future work on the project is best focused on uncertainty quantification, especially with comparison to industry practice such as double-lattice models as well as potential flight or wind tunnel testing. Further research is also needed to determine the effects of anisotropic materials such as fiberglass or carbon fiber, which are common in vehicle construction. Establishing guidelines for damping coefficients, accounting for compressibility effects, and improving calculation of torsion and bending mode frequencies.

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References

[1] Martin, D. J., "Summary of Flutter Experiences as a Guide to the Preliminary Design of Lifting Surfaces on Missiles," Tech. Rep. TN-4197, NACA, 1958. URL https://ntrs.nasa.gov/citations/19930085030.

[2] Bennet, J. K., "Fin Flutter Analysis Revisited (Again)," Tech. rep., Nakka-Rocketry.net, 2023. URL https://www.nakka-rocketry.net/articles/Calculating_Fin_Flutter_Velocity_Bennett-12-23.pdf.

[3] Theodorsen, T., "General Theory of Aerodynamic Instability and the Mechanism of Flutter," Tech. Rep. TR-496, NACA, 1949. URL https://ntrs.nasa.gov/citations/19930090935.

[4] Theodorsen, T., and Garrick, I. E., "Mechanism of Flutter A Theoretical and Experimental Investigation of the Flutter Problem," Tech. Rep. TR-685, NACA, 1940. URL https://ntrs.nasa.gov/citations/19930091762.

[5] Weisshaar, T. A., *Aeroelasticity: A One-Semester Course*, Self-Published, 2022.

[6] Fung, Y. C., *An Introduction to the Theory of Aeroelasticity*, Dover, 1993.

[7] Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Dover, 1996.

[8] Simmons, J. R., "AEROELASTIC OPTIMIZATION OF SOUNDING ROCKET FINS," Master's thesis, Air Force Institute of Technology, 2009. URL https://ntrl.ntis.gov/NTRL/dashboard/searchResults/titleDetail/ADA502110.xhtml.

[9] Kearns, J. P., "Flutter Simulation," Tech. rep., The John Hopkins University Applied Physics Laboratory, 1962. URL https://apps.dtic.mil/sti/citations/AD0650981.

[10] Avitabile, P., “Experimental Modal Analysis (A Simple Non-Mathematical Presentation),” *Sound and Vibration Magazine*, 2001.