A Practical Approach to Plate Solving of Astronomical Images Based on Least Squares Estimation

Cindy Nguyen[1] and Thomas Alan Lovell[2]
Embry-Riddle Aeronautical University, Daytona Beach FL

This method presents a method of plate solving (i.e. registration) of an unresolved space image in order to determine the inertial direction and orientation of the image. Fundamentally, the strategy involves performing edge detection and centroiding on the image to estimate the location of each illuminated object in the image, then attempting to match star patterns in the image with those from a known database (i.e. star catalog). Rather than storing a large volume of template images, the method converts both the image centroids and catalog stars to unit vectors to determine the appropriate camera-to-inertial coordinate transformation for the image.

I. Introduction

A common task in astrophotography (i.e. imaging of space) is image registration, also referred to as plate solving. Broadly speaking, this is the task of determining the direction and orientation of an astronomical image with respect to the celestial sphere based on recognition of star patterns found in the image. The development of plate solving tools is generally in the domain of astronomers, and the most widely used method for the past several years is an application known simply by the name Astrometry [1]. Plate solving is also required to determine a spacecraft's attitude when utilizing a star tracker camera, and therefore a number of star tracker manufacturers have developed their own proprietary techniques.

Most plate solving methodologies involve comparing stars in an image with a known database of simulated template images that are developed based on a variety of assumed fields of view, from narrow (e.g. 0.5 degrees or less) to wide (e.g. 20 degrees or greater). A typical plate solving run involves a search through the image database to find a match among asterisms (i.e. groups of stars) in the real image (which could be construed as mini-constellations). The runtime of this search can typically be improved if the approximate direction and orientation of the image are already known. Otherwise, a blind search proceeds, which can incur significant runtime. Depending on the platform, this could be upwards of several minutes, which is unacceptable for many realistic applications, such as processing a large number of images for various purposes.

In this paper the authors develop an alternate approach to plate solving compared to those in current use or in the literature. This approach assumes the camera's focal length and pixel size are known. The proposed procedure is as follows:

- Given a space image, detect and localize each object (i.e. star) in the camera frame in terms of local (x,y) pixel coordinates (e.g. right-down-forward)
- For each star detected, combine its (x,y) location with the above-mentioned camera parameters to calculate the unit vector from the camera (i.e. observing platform) to the star, expressed in the local coordinate frame
- Compare the juxtaposition of unit vectors of the stars in the image with those contained in a standard star catalog (e.g. Tycho-2) to eventually yield a least-squares solution of the coordinate transformation (in terms

---

[1] Undergraduate Student, Aerospace Engineering Department, AIAA Student Member 1762168.
[2] Professor, Aerospace Engineering Department, AIAA Associate Fellow.

of a direction cosine matrix) between the local camera frame and the inertial frame of the star catalog (e.g. J2000)[2]

This last step will be achieved by dividing the star catalog into sectors of right ascension and declination (RA and Dec), where the size of each sector in degrees will be roughly equal to the image's known field of view. Proceeding one sector at a time, a comparison between the stars in an image and those in the star catalog can be conducted by solving the over-determined attitude navigation problem (more commonly known as Wahba's problem[3]) and calculating the residuals in right ascension (RA) and declination (Dec) between the two sets of unit vectors. A "match" can then be assessed based on how close the residuals are to some chosen threshold in degrees.

II. Coordinate Transformations

Fundamentally, plate solving an image equates to determining the relationship between the orientation of the camera frame at the time instant of the image and the orientation of the reference frame of a star catalog, where the latter is typically an Earth-centered inertial (ECI) frame such as J2000. Thus, a plate solving solution itself normally consists of a coordinate transformation between these two frames. The Astrometry tool returns a solution in terms of RA, Dec, and θ, where the RA and Dec values indicate the inertial (i.e. celestial) direction in which the camera's boresight was pointing at the time of the image, and θ is the angle between the local "up" direction of the image and celestial North. As shown below, this set of values in fact follows the construction of Euler angles.

A. Direction Cosine Matrix

It is well known that a direction cosine matrix (DCM) denoted Q relates the unit vectors of one coordinate frame to those of another coordinate frame. Therefore, we can write:

$$\hat{u}_{J2K} = \boldsymbol{Q}_{\boldsymbol{CAM}}^{\boldsymbol{J2K}} \hat{u}_{CAM} \tag{1}$$

Where $\hat{u}_{CAM}$ represents a unit vector in the camera's coordinate frame and $\hat{u}_{J2K}$ represents a unit vector in the J2000 frame. If we are given $\hat{u}_{CAM}$ and $\hat{u}_{J2K}$ and wish to solve $\boldsymbol{Q}_{\boldsymbol{CAM}}^{\boldsymbol{J2K}}$ there is not a unique solution, i.e. there is an infinite number of Q transformations that will solve Equation (1). However, if we have two unit vectors whose coordinates in both the camera frame and J2000 are known, then a unique transformation exists. The preceding statements assume that the unit vectors are perfectly known, i.e. with no error. In a real scenario, this is not the case, and therefore it is common to utilize as many unit vectors one can obtain whose coordinates in both frames are (imperfectly) known.

While the process of plate solving follows the above paradigm, there are other applications in aeronautics, astronautics, robotics, and other fields. In particular, the problem of spacecraft attitude determination has received significant attention over the years. Several researchers in this field have derived algorithms to solve the over-determined problem commonly known as Wahba's Problem[3], which in the context of plate solving can be stated as:

$$J = \sum_k \left| \hat{u}_k|_{J2K} - \boldsymbol{Q}_{\boldsymbol{CAM}}^{\boldsymbol{J2K}} \hat{u}_k|_{CAM} \right| \tag{2}$$

That is, solve the direction cosine matrix that minimizes the residuals between the two sets of unit vectors.

B. Euler Angles

In the sense that a direction cosine matrix represents a rotation from one coordinate frame into another, it is well known that the transformation between any two coordinate frames of arbitrary orientation can be represented as a sequence of elementary (*x*, *y*, or *z*-axis) rotations. The maximum number of elementary rotations required to represent a given transformation is 3, and furthermore we can choose any sequence of elementary rotations we wish (known as the Euler sequence) to represent the overall transformation. The angles involved in each of these rotations are known as Euler angles. For a given Euler sequence, the Euler angles are usually referred to in order as α, followed by β, followed by γ.

The Euler sequence we will emphasize here is the 3-2-3 sequence, consisting of a rotation about the *z*-axis, followed by the *y*-axis, and finally the *z*-axis. This particular sequence relates closely to the RA, Dec, and θ values alluded to above, in the following way. Suppose we define the camera's coordinate frame as having the origin at the camera boresight, *x* to the right, *y* in the "down" direction, and *z* in the "forward" direction. Combined with the familiar definition of the J2000 frame (i.e. origin at Earth's center, *x* toward the vernal equinox as of noon UTC on 1 Jan 2000, *z* toward Earth's axis of rotation, and *y* completing the orthogonal triad), Figure 1 below illustrates the Euler sequence from the camera frame (denoted with subscript "c") to J2000 (denoted with subscript "ECI"). Noting that the RA, Dec, and θ values are indicated in the figure, we see that $\alpha = -(90° + \theta)$, $\beta = 90° -$ Dec and $\gamma = 180 -$ RA.

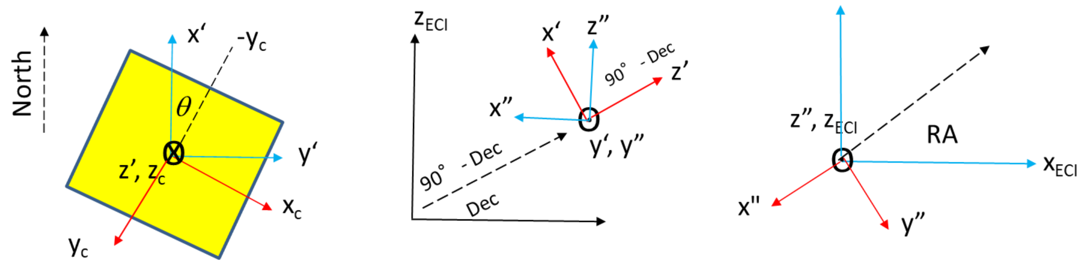

Figure 1 Illustration of the 3-2-3 Euler sequence, as applied to the camera frame to J2000 transformation.

If we write the DCM for each rotation in the Euler sequence above [2] and multiply these three matrices together, we obtain the overall DCM between the camera frame and J2000 expressed as functions of α, β, and γ:

$$\boldsymbol{Q}_{CAM}^{J2K} = \begin{bmatrix} cos\gamma cos\beta cos\alpha - sin\gamma sin\alpha & cos\gamma cos\beta sin\alpha + sin\gamma cos\alpha & -cos\gamma sin\beta \\ -sin\gamma cos\beta cos\alpha - cos\gamma sin\alpha & -sin\gamma cos\beta sin\alpha + cos\gamma cos\alpha & sin\gamma sin\beta \\ sin\beta cos\alpha & sin\beta sin\alpha & cos\beta \end{bmatrix} \quad (3)$$

Thus, if we know the elements of the DCM, we can solve for α, β, and γ as follows:

$$\beta = cos^{-1}(Q_{33}), \alpha = tan^{-1}\left(\frac{Q_{32}}{Q_{31}}\right), \gamma = tan^{-1}\left(\frac{Q_{23}}{-Q_{13}}\right) \quad (4)$$

where, for α and γ we must be mindful of the quadrant in which the answer resides, based on the sign of the numerator and denominator.

III. Plate Solving Methodology

Looking back at Equation (1), suppose we have three unit vectors whose coordinates in both the camera frame and J2000 are known. Denote these vectors $\hat{u}_1|_{CAM}$, $\hat{u}_2|_{CAM}$, and $\hat{u}_3|_{CAM}$ in the camera frame and $\hat{u}_1|_{J2K}$, $\hat{u}_2|_{J2K}$, and $\hat{u}_3|_{J2K}$ in J2000. Then it is true that:

$$\boldsymbol{Q}_{CAM}^{J2K}[\hat{u}_1|_{CAM} \quad \hat{u}_2|_{CAM} \quad \hat{u}_3|_{CAM}] = [\hat{u}_1|_{J2K} \quad \hat{u}_2|_{J2K} \quad \hat{u}_3|_{J2K}] \quad (5)$$

so that:

$$\boldsymbol{Q}_{CAM}^{J2K} = [\hat{u}_1|_{J2K} \quad \hat{u}_2|_{J2K} \quad \hat{u}_3|_{J2K}][\hat{u}_1|_{CAM} \quad \hat{u}_2|_{CAM} \quad \hat{u}_3|_{CAM}]^{-1} \quad (6)$$

This method of solving *Q* is often referred to as the TRIAD method. Equipped with this technique, the proposed plate solving methodology is here described below.

A. Initial Data

For a typical plate solving scenario, we assume the initial data consists of a raw image taken by a camera whose focal length and pixel size are known. The database of "true" star locations to compare to would consist of a reputable catalog such as Tycho-2.[4] These locations are typically stored as RA and Dec, but can be easily converted to unit vectors expressed in the J2000 frame by:

$$\hat{u}_{J2K} = \begin{bmatrix} \cos(Dec)\cos(RA) \\ \cos(Dec)\sin(RA) \\ \sin(Dec) \end{bmatrix} \quad (7)$$

B. Thresholding, Edge Detection, and Centroiding of an Image

Fundamentally, a raw monochrome image consists of a matrix in which the intensity (i.e. brightness) value of each pixel in the focal plane (i.e. detector array) is stored. Each pixel contains a certain degree of noise, i.e. random fluctuations in intensity, typically resulting from electronic phenomena. B. Thresholding involves setting a lower intensity limit such that all pixels whose intensity falls below this limit will be assigned an intensity value of zero and therefore be considered black sky. Edge detection is then a gradient-based scheme to determine the boundary of each collection of contiguous illuminated pixels, i.e. each "object" (most if not all of which are stars). The centroid of each object is then calculated as follows:

$$x_c = \frac{\sum_{i=1}^{N} x_i w_i}{\sum_{i=1}^{N} w_i},\ y_c = \frac{\sum_{i=1}^{N} y_i w_i}{\sum_{i=1}^{N} w_i} \quad (8)$$

Here x_i corresponds to the local x coordinate of the ith pixel in the image, in terms of number of pixels to the left or right of the boresight (positive to the right). Similarly, y_i corresponds to the local y coordinate of the ith pixel in the image, in terms of number of pixels above or below the boresight (positive down). Also, w_i corresponds to the intensity of the ith pixel.

C. Generating Unit Vectors in the Camera Frame

Note that the coordinates of each centroid, x_c and y_c above, are in pixel units rather than physical units (i.e. length). However, these can be converted to physical units simply by multiplying by the pixel size (usually in mm). If the pixel dimensions are ε x ε and the camera's focal length is f, then the vector from the focal point of the camera's lens to the centroid's location in the focal plane expressed in camera frame coordinates is then:

$$\bar{r}_{CAM} = \begin{bmatrix} \varepsilon x_c \\ \varepsilon y_c \\ f \end{bmatrix} \quad (9)$$

Note that this vector is proportional to the vector from the camera out to the actual star represented by the centroid. Therefore, if we divide this vector by its magnitude, we obtain the unit vector to the star in camera frame coordinates:

$$\hat{u}_{CAM} = \begin{bmatrix} \frac{\varepsilon x_c}{L} \\ \frac{\varepsilon y_c}{L} \\ \frac{f}{L} \end{bmatrix} \quad where \quad L = \sqrt{\varepsilon x_c{}^2 + \varepsilon y_c{}^2 + f^2} \quad (10)$$

The plate solving method presented here involves comparing camera frame unit vectors calculated from image centroids via Equation (10) with J2000 unit vectors calculated from a star catalog via Equation (7).

D. Pairing Process

The actual plate solving strategy proceeds as follows:

- Divide the star catalog into RA and Dec "sectors," whose size is based on the camera's field of view. For example, for a camera with a 2° x 2° field of view, given that RA varies from 0° to 360° and Dec varies from -90° to 90°, the catalog will be divided into 180 * 90 = 16,200 sectors 2° x 2° in size.
- For each sector, choose the camera frame unit vectors corresponding to the p brightest stars in the image and the J2000 unit vectors corresponding to the q brightest stars in the catalog.
- Choose a triplet of 3 camera frame unit vectors from p total vectors, and similarly for the J2000 unit vectors. Choosing a particular order for each triplet, solve the DCM (designated as $\boldsymbol{Q}_{CAM}^{J2K}$) based on Equation (6) above. Then convert to Euler Angles based on Equation (4) above.

- Continue this process for all *p*-choose-3 triplets of camera frame unit vectors paired with all *q*-choose-3 triplets of J2000 unit vectors, allowing for each triplet to appear in any possible order. Since a set of three unique numbers can be ordered in 6 different ways, the total number of pairings (i.e. the total number of DCM and Euler Angle solutions) will be $\xi = (p\text{-choose-}3)*6*(q\text{-choose-}3)*6$
- Plot a histogram of the Euler Angles solutions (ξ values of α, β, and γ) and look for aggregation among the solutions, i.e. a small standard deviation. When the correct sector has been found, it is expected that the standard deviation among the solutions in this sector will be much smaller (i.e. the solutions will be more "tightly packed") than in any other sector of the celestial sphere.

IV. Results

The above-described algorithm was tested on the image shown in Figure 2, which was taken with a Celestron RASA 11-inch telescope with an ASI 1600mm monochrome CMOS camera mounted on a Celestron CGE Pro mount. While a full-scale blind search has not yet been conducted, the first step has been to plate solve the image using the Astrometry tool and utilize the output in the following way:

- Obtain the output file titled "*corr.fits*". This contains the list of centroids in the image that were matched with stars in the catalog; next to each centroid, the RA and Dec of its corresponding catalog star is listed.
- For the first 10 stars in the list, convert each image centroid into a camera frame unit vector using Equation (10) and convert each catalog RA and Dec into a J2000 unit vector using Equation (7).
- CASE 1: Form all possible 10-choose-3 triplets of camera frame unit vectors and J2000 unit vectors, and for each camera frame triplet, pair it with its "true" counterpart, i.e. pair the 1^{st}, 4^{th}, and 7^{th} camera frame unit vectors with the 1^{st}, 4^{th}, and 7^{th} J2000 unit vectors. For each pair of triplets, solve the DCM via Equation (6) then convert to Euler Angles. This produced the histograms shown in Figures 3-5.
- CASE 2: Next, perform all possible pairings between the camera frame unit vectors and J2000 unit vectors, allowing for the order of vectors to be different (the total number is (10-choose-3)*6*(10-choose-3)*6). For each pair of triplets, solve the DCM via Equation (6) then convert to Euler Angles. This produced the histograms shown in Figures 6-8.

First, the histograms of Figures 3-5 generally show consistency among the solutions, i.e. that nearly all triplets of camera frame unit vectors undergo nearly the same coordinate transformation into their counterpart triplets of J2000 unit vectors. Bear in mind that these solutions involve each camera frame triplet being paired with its known correct (according to Astrometry) counterparts in the J2000 frame. The histograms of Figures 6-8, however, involved mostly incorrect pairings, yet aggregation is still seen for β and γ. While further study of this technique is definitely warranted, it shows promise that the correct sector of the celestial sphere could be found (essentially the pointing direction of the boresight), while solving the correct value of α would amount to a 1-D search process.

References

[1] Lang, D. W. Hogg, K. Mierle, M. Blanton, and S. Roweis, "Astrometry.net: Blind astrometric calibration of arbitrary astronomical images," Oct. 2009.
[2] Curtis, D., "Orbital mechanics for engineering students," 4th edition.
[3] Shuster, M.D., "The generalized Wahba problem," The Journal of the Astronautical Sciences, 2006
[4] Hog E., Fabricius C., Makarov V.V., Urban S., Corbin T., Wycoff G., Bastian U., Schwekendiek P., Wicenec A., "The Tycho-2 Catalogue of the 2.5 Million Brightest Stars," 2000.

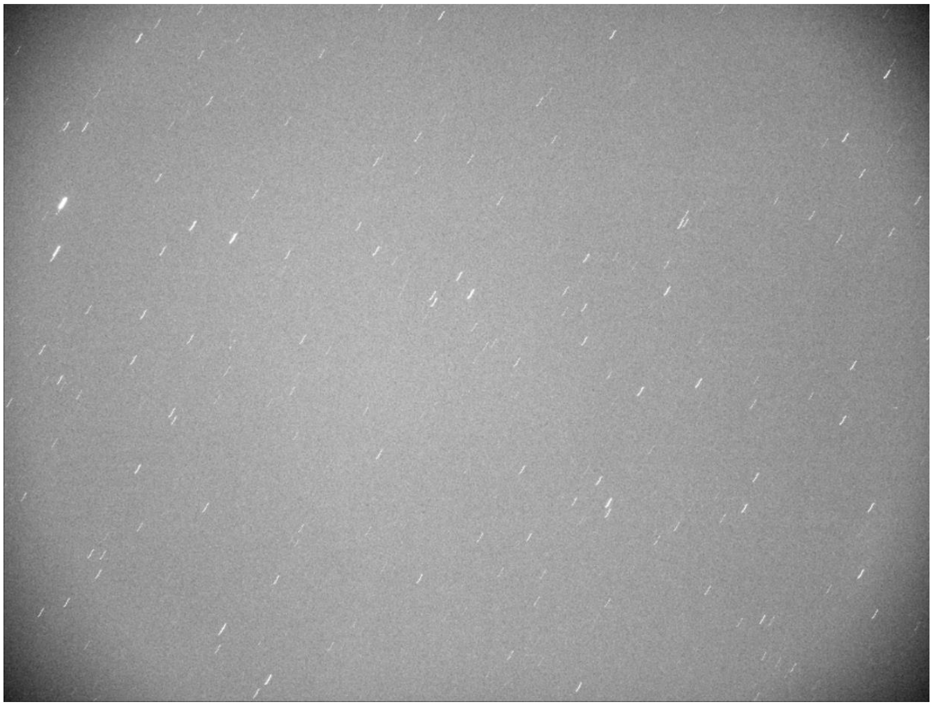

Figure 2 Test image for plate solving algorithm.

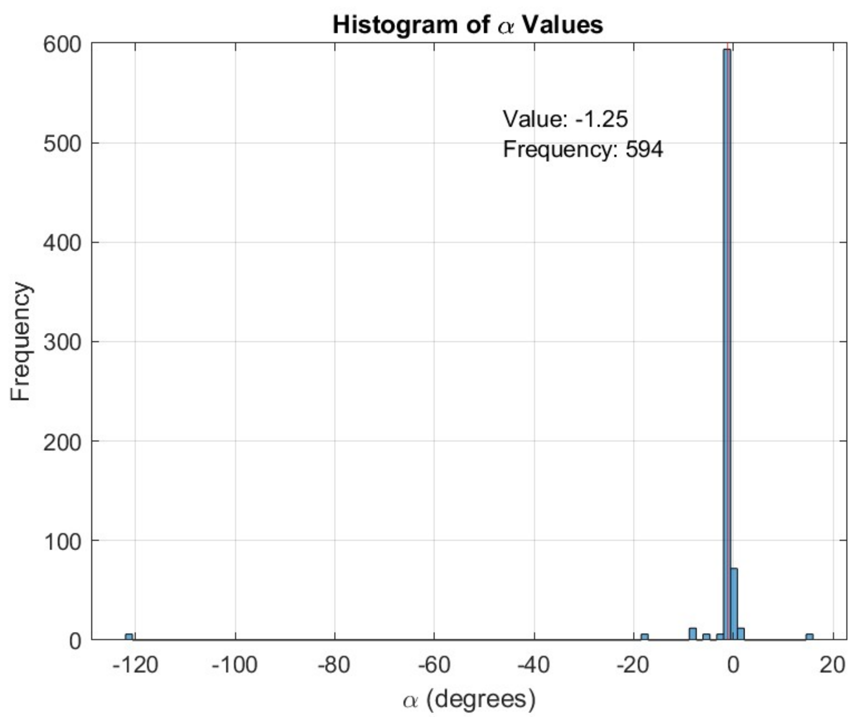

Figure 3 Histogram of α Euler angle solution (CASE 1).

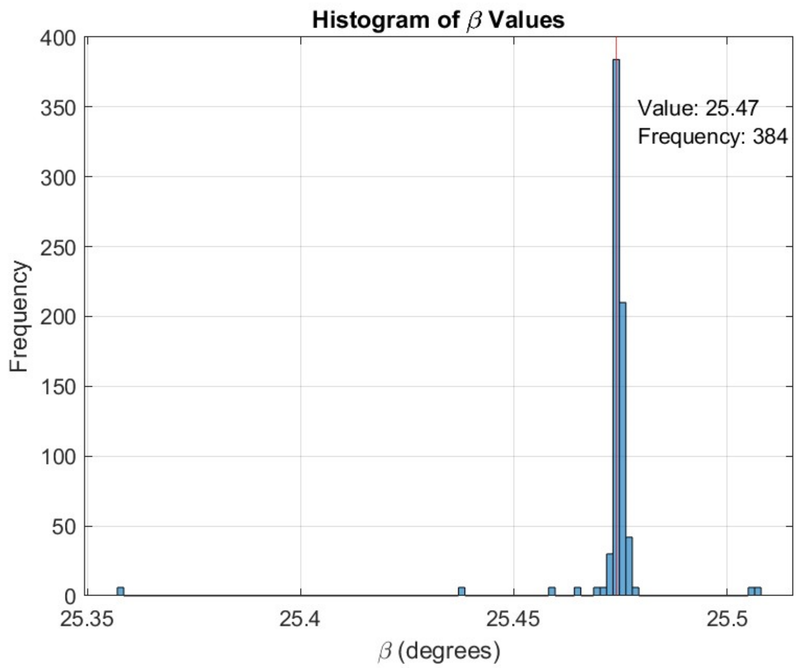

Figure 4 Histogram of β Euler angle solution (CASE 1).

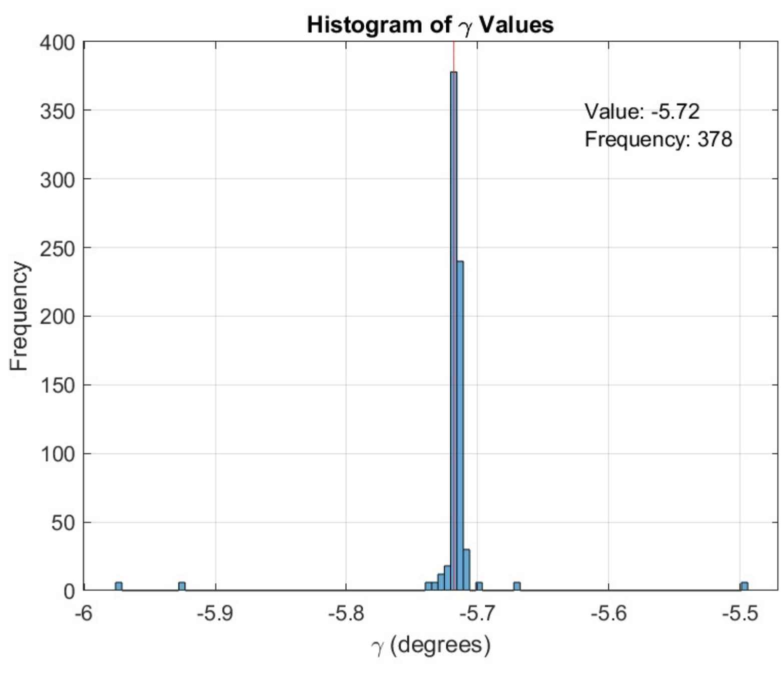

Figure 5 Histogram of γ Euler angle solution (CASE 1).

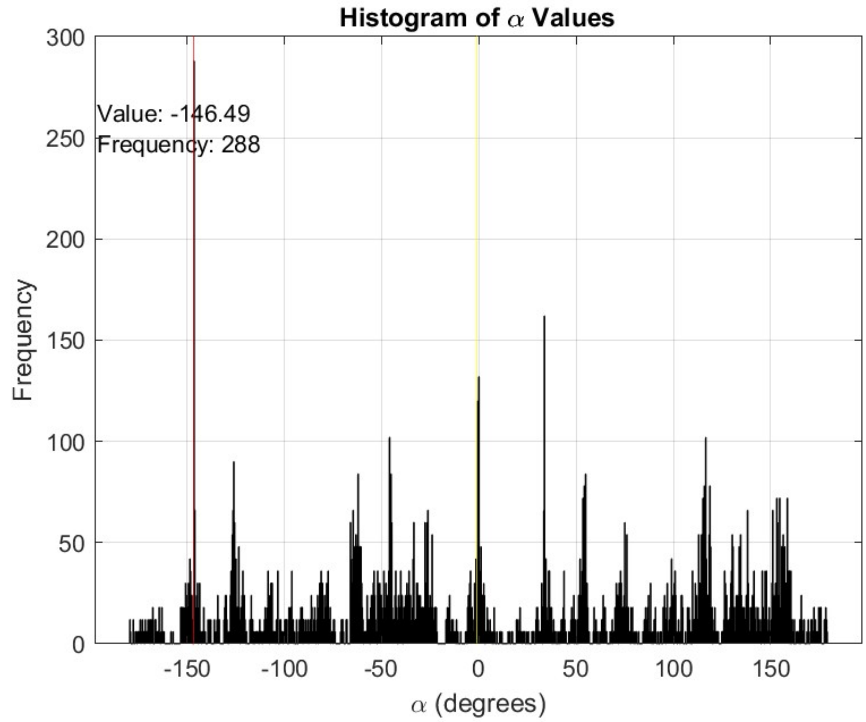

Figure 6   Histogram of α Euler angle solution (CASE 2).

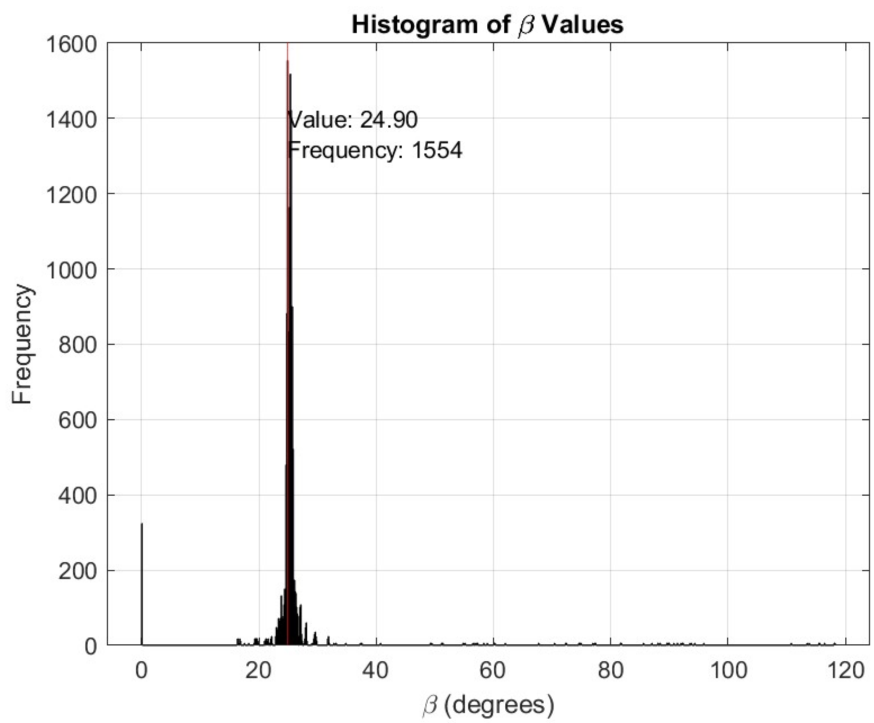

Figure 7   Histogram of β Euler angle solution (CASE 2).

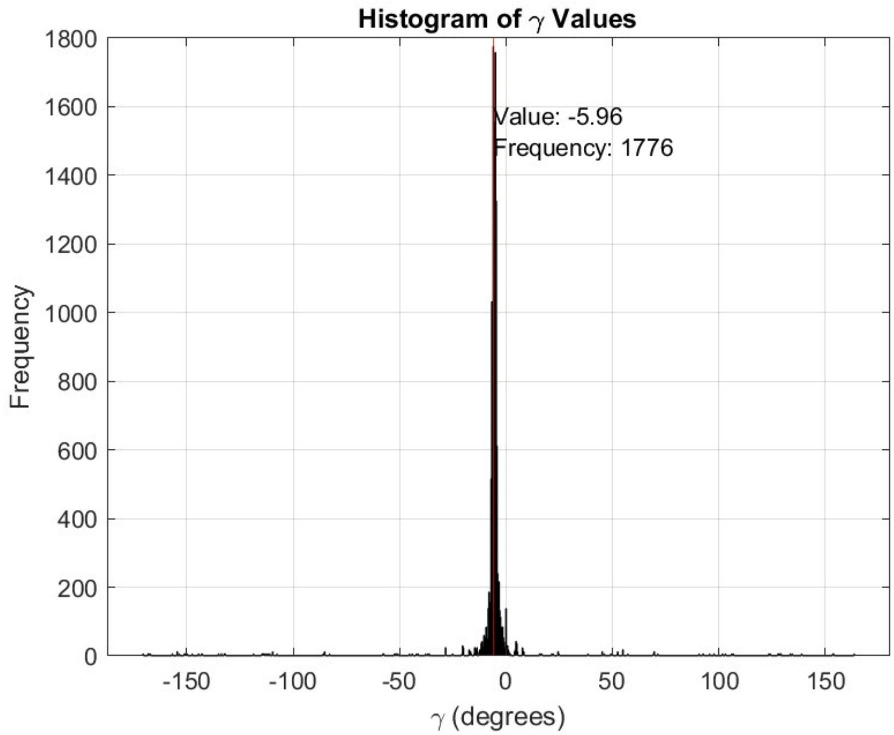

Figure 8 Histogram of γ Euler angle solution (CASE 2).