The Application of Phase Sensitive Detection in High-Resolution Acoustic Measurements

Tymur Tkachenko* and Shawn Wehe†

Georgia Institute of Technology, Atlanta, Georgia, 30332

The study of combustion dynamic processes commonly involves the use of acoustic signals and measurements of flame response. Combustion dynamics describes a positive feedback effect in combustion systems where there is a cycle of heat release, velocity, and pressure fluctuations in a combustor flow field. If left unchecked, pressure amplitudes that result from the feedback can catastrophically damage a combustion system like a rocket motor or gas turbine combustor. Currently, the standard is to use fixed-frequency measurements using acoustic forcing from speakers or sirens. There is a natural trade-off between the resulting frequency resolution and the time required to complete a frequency sweep over a bandwidth of interest. Incomplete spectral information is obtained if the frequency steps are too large but long experimental runs are required if a high resolution result is desired. The trade-off results in greater expense associated with additional compressor, fuel, and labor costs for high-frequency resolution measurements. This work demonstrates a method to replace fixed-frequency measurements with a chirped-frequency technique, allowing for faster measurements and nearly continuous frequency resolution. Two experiments are discussed to demonstrate the technique in simple, room-temperature experiments. The first is a transfer function measurement on a sense tube which is normally used in combustion experiments. The sense tube protects a delicate pressure transducer from elevated temperatures by installing the transducer on a standoff tube away from a combustion chamber. The second experiment discusses the effects of acoustic impedance from the endwall of an acoustic chamber. In the first experiment, the traditional fixed-frequency measurement is directly compared to the chirped-frequency technique. In the second experiment, an experimental setup and procedure is discussed.

I. Nomenclature

A	=	amplitude of oscillation
f	=	frequency of oscillation
$\frac{df}{dt}$	=	rate of change of frequency
T	=	period of oscillation
ω	=	angular frequency of oscillation
ϕ	=	phase of oscillation
α, β	=	generic angles
t	=	time
$\mathrm{d}t$	=	time step
x, y	=	generic functions
p	=	pressure
L	=	length
c	=	speed of sound
Z	=	acoustic impedance
ρ	=	density
k	=	wave number
r	=	acoustic reflection factor
i, j	=	imaginary constant

*Undergraduate Researcher, Department of Electrical and Computer Engineering, ttkachenko3@gatech.edu, and AIAA Student Member.

†Principal Research Engineer, Department of Aerospace Engineering, swehe7@gatech.edu.

II. Introduction

In the testing of combustion dynamic processes, the use of acoustic signals to measure flame response is commonplace. Currently, the standard is to use fixed-frequency measurements. There is a trade-off between the time it takes to take fixed-frequency measurements and the resulting frequency resolution. A greater time to take measurements results in greater costs in the form of fuel and labor, while an incomplete frequency resolution results in gaps in data. A chirped-frequency measurement, on the other hand, would be able to provide a near continuous frequency resolution in the same time as a fixed-frequency measurement. Chirped-frequency measurements, or swept-sine measurements, have previously been applied for measuring acoustics in large rooms such as concert halls by measuring the acoustic response in different locations and then processing the data [1]. This paper aims to demonstrate a proposed method to apply a similar technique to acoustic combustion dynamic measurements.

The experimental setup consists of a tube with a speaker on one end and the other end open. The tube has two pressure transducers located at the open end, one connected directly into the tube, and another located far from the tube on a semi-infinite sense tube. During combustion measurements, pressure transducers must be located far away from the flame to avoid damage, however, the further the signal must travel, the more it will attenuate. A transfer function creates a map from the pressure recorded on the sense tube and the pressure inside the combustion chamber. The experimental setup recreates a typical transfer function combustion measurement. In fixed-frequency measurements, amplitude of the pressure response was determined via an analog lock-in amplifier. A lock-in amplifier is a device that can be used to determine a periodic signal's amplitude in narrow bandwidth and is effective in noisy environments.

A method of determining the amplitude of a signal resulting from a source that is swept through a range of frequencies (i.e, a chirped signal) was developed by using a reference phase technique. A reference phase method uses the integral of frequency to create a reference phase signal which can be used in a digital lock-in amplifier algorithm to determine the amplitude of a chirped-frequency signal. The fixed-frequency measurements taken using a lock-in amplifier and the chirped-frequency results processed digitally were directly compared and showed significant similarities.

For future measurements, a tube with a perforated end will be used to compare fixed-frequency and chirped measurements of acoustic impedance. Furthermore, the tube will be instrumented in such a way as to create conditions in which the frequency of the signal travelling through it cannot be assumed to be constant. These spectral conditions will be achieved by modifying the chirp's rate of change of frequency, tube length and instrumentation location, and speed of sound inside the tube.

The following sections will discuss the setup, methodology, and results of the two experiments. Section III will discuss the design of the transfer function experiment and the necessary equipment. It will also cover the methodology in acquiring and processing data, the architecture of lock-in amplifiers, and the digital algorithm (ULIA) for processing chirped-frequency signals. It also covers measures taken to reduce electrical noise in the systems. Section IV will cover processed data from the transfer function experiment and discuss the results, including topics such as filters, errors, and various chirp durations. Section V will discuss specific design and analysis changes necessary for acoustic impedance measurements. Section VI will conclude the findings and discuss future research.

III. Design and Methodology of a Transfer Function Experiment

For the transfer function experiment, a straight tube with a nonporous inner surface is used. The tube has a speaker, an open end, and two acoustic sensors. The tube used was also able to vary in length. The speaker is driven by a function generator audio-amplifier combination to create chirped acoustic signals. In the fixed-frequency setup, data is collected with the acoustic sensors that connect to a signal conditioner. The signal from the conditioner is then processed by a lock-in amplifier. The function generator sends a reference signal of the same frequency as the driving frequency of the speaker to the lock-in amplifier. In chirped-frequency experiments, the lock-in amplifier is removed and the signal from the conditioner is routed to an oscilloscope. Data is recorded on the oscilloscope and processed on a computer. The experiment is largely inspired by Boyle, Henderson, and Hultgren's experiments [2]. The physical setups in both variations are detailed in Fig. 1. Once data is collected, a transfer function can be determined by finding the ratio between transducer 2 and transducer 1, and the difference in phase between the two.

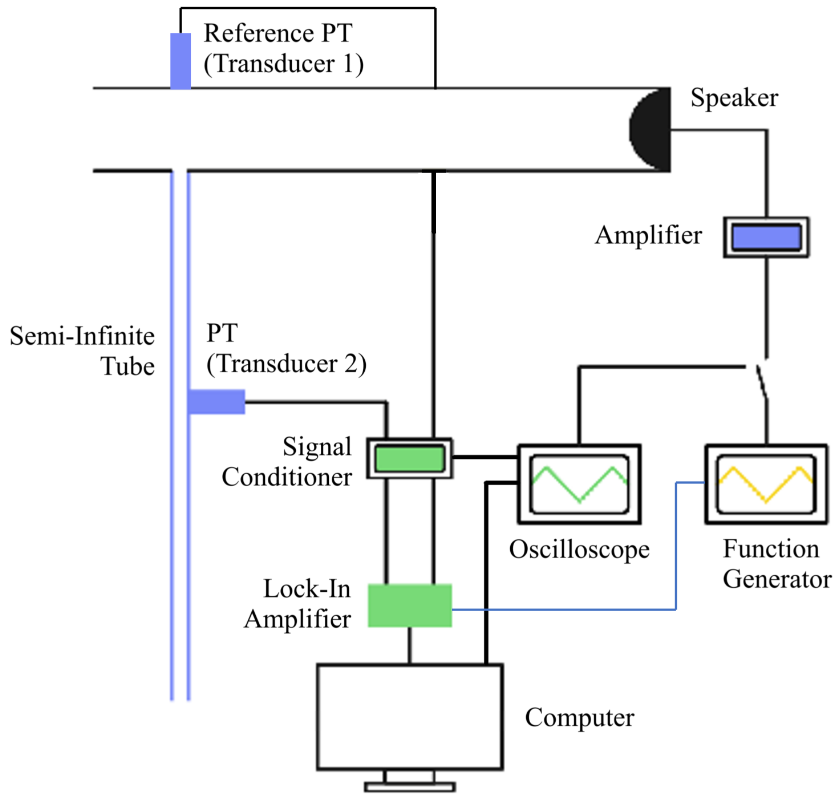

Fig. 1 Diagram of transfer function experimental setup.

The transducer labeled "Reference PT" is henceforth referred to as "Transducer 1" and "PT" is referred to as "Transducer 2" in this paper.

A. Fixed Frequency Measurements

Fixed-frequency measurements were acquired between the range of 300 and 2500 Hz. First, a measurement was taken every 200Hz. An interval of 200 Hz was not enough to adequately define the peaks, so more measurements were taken in intervals of 10 Hz between 600 Hz and 800 Hz. Measurements were taken by adjusting the frequency of the signal generated by the function generator, as well as the frequency of the reference signal provided to the lock-in amplifier. The lock-in amplifier sensitivity was optimized to maximize the signal resolution. The value from the lock-in amplifier was then recorded on a computer.

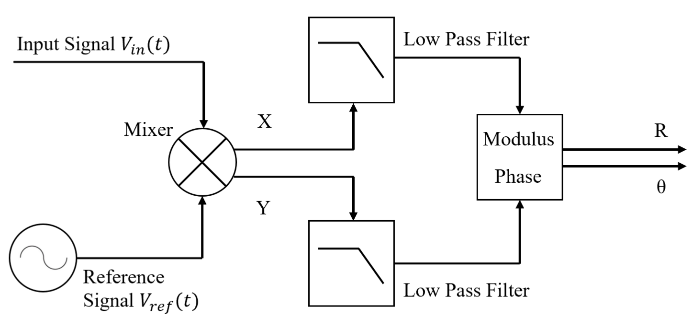

Fig. 2 Simplified block diagram of analog lock-in amplifier.

B. Chirped Frequency Measurements

Chirped frequency measurements were taken by programming a function generator with a chirped signal, going from 300 Hz to 2500 Hz over periods of 0.001, 0.020, 0.100, 0.200, 0.500, 1, 2, 5, 10, and 20 seconds. Data was recorded on an oscilloscope at a sampling frequency of 10 kHz and processed in MATLAB. An unsteady lock-in amplifier algorithm (ULIA) was developed by applying concepts from an analog lock-in amplifier and applying a reference phase method.

A reference phase method was applied to synthesize a reference signal with a chirped-frequency, where ϕ_{ref} is the integral of the reference frequency f_{ref}.

$$\phi_{ref}(t) = \int_{t_0}^{t_1} 2\pi f_{ref}(t)\ dt = \int_{t_0}^{t_1} \omega_{ref}(t)\ dt \tag{1}$$

A time-varying reference sine and cosine were then synthesized using the reference phase.

$$\sin_{ref}(t) = \sin(\phi_{ref}(t)) \tag{2}$$

$$\cos_{ref}(t) = \cos(\phi_{ref}(t)) \tag{3}$$

The recorded signal from the tube can be written in the form

$$\sin_{sig}(t) = A(t)\sin(2\pi f_{sig}(t) + \phi_{sig}) \tag{4}$$

Multiplying the sine and cosine with the recorded signal results in two new signals following the trigonometric identities:

$$A(t)\sin(\alpha)\cos(\beta) = A(t)(\frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2}) \tag{5}$$

$$A(t)\sin(\alpha)\sin(\beta) = A(t)(\frac{\cos(\alpha+\beta) - \cos(\alpha-\beta)}{2}) \tag{6}$$

Multiplying equation (4) by equations (2) and (3) results in two components of the original signal. The components can then be multiplied by a factor of 2 to compensate for the 1/2 in the trigonometric identities.

$$x_1 = A(t)(\sin(2\pi f_{sig}(t) + \phi_{sig} + \phi_{ref}(t)) + \sin(2\pi f_{sig}(t) + \phi_{sig} - \phi_{ref}(t))) \tag{7}$$

$$y_1 = A(t)(\cos(2\pi f_{sig}(t) + \phi_{sig} + \phi_{ref}(t)) - \cos(2\pi f_{sig}(t) + \phi_{sig} - \phi_{ref}(t))) \tag{8}$$

A low pass filter with a a cut-off frequency can then be applied to these equations to remove the higher frequency term, resulting in two sinusoids near DC (0 Hz). For this experiment, a cutoff frequency of 2 Hz was selected.

$$x_1 = A(t)(\sin(2\pi f_{sig}(t) + \phi_{sig} - \phi_{ref}(t))) \tag{9}$$

$$y_1 = A(t)(\cos(2\pi f_{sig}(t) + \phi_{sig} - \phi_{ref}(t))) \tag{10}$$

Ideally, the resulting signals will be close to 0 Hz; the 2-norm of the resulting signals will give the amplitude of the original signal.

$$A(t) = \sqrt{x_1^2 + y_1^2} \tag{11}$$

The fixed-frequency measurements were then directly compared to the results from the ULIA.

C. Electrical Noise

Significant background electrical noise was recorded in the signal from the tube. Frequencies of noise at 60 Hz with harmonics at 120 Hz and 180 Hz heavily overpowered the frequency of the chirp at lower speaker amplitudes. 60 Hz is a common frequency of electrical noise in the US because it is the frequency of the electrical grid. These frequencies were filtered out of the recorded signal by applying a notch filter.

IV. Results and Discussion of the Transfer Function Experiment

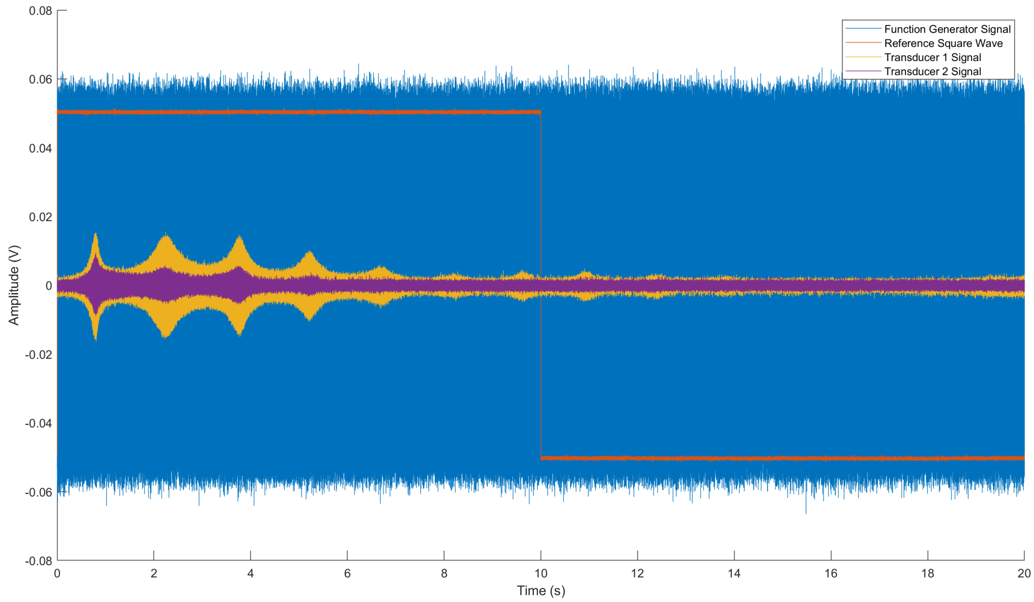

Fig. 3 Raw signal data in a 20 second chirp experiment.

Figure 3 displays raw data from the function generator two pressure transducers, and a reference square wave. Transducer 1 is referencing the transducer directly on the tube, whereas transducer 2 is placed on a sense tube. The amplitude of the signal from the function generator was set to 100 mVpp in this experiment. The figure shows peaks in amplitude recorded in both transducers signals at the same points in time. Eventually both stop having a significant response at frequencies greater than 1.2 kHz.

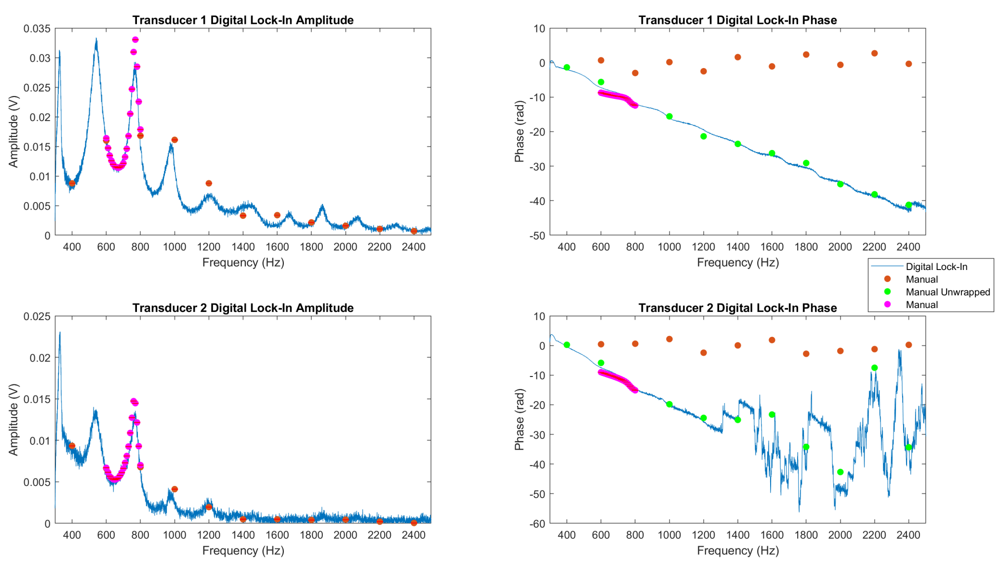

Fig. 4 ULIA results compared with fixed-frequency data in a 5 s chirp.

Figure 4 shows the fixed-frequency measurements collected as described in section placed over the results of the ULIA algorithm. The chirp duration was 5 seconds over a frequency range of 300 to 2500 Hz. The error bars were recorded as ± 1 in the last significant digit and are smaller than the data symbols. The fixed-frequency measurements, especially the ones in the 600-800 Hz range, can be seen to closely follow the ULIA until the peak, when the fixed-frequency measurements become much steeper. A suspected cause of this discrepancy is the speed of the chirp. That is, the steady-state signal accurately depicts the sharp signal amplitude response through the peak with changing speaker frequency. However, during the chirped experiment, the frequency axis actually sweeps in time. Therefore sharp peaks in the sensor response likely have frequency components beyond the low pass filter in the ULIA. A Fourier expansion can be performed on the fixed frequency measurements, assuming the frequency axis is changing in time as in the chirped experiments as shown in figure 5. The Fourier expansion yielded a natural frequency of 8 Hz for the steepness of the fixed-frequency data, far above the selected low pass filter of 2 Hz. However, when increasing the frequency of the filter, the resulting signal will become noisier. A potential solution is to decrease the $\frac{df}{dt}$ of the chirp, bringing the frequency of the peaks into the ULIA lowpass. The phase of the signal in transducer 1 is seen to be linear with frequency for the range. However, going into the upper half of the frequency range, the signal-to-noise ratio increases and it becomes difficult to accurately determine the phase of transducer 2. Even after being unwrapped, the phase is highly unstable.

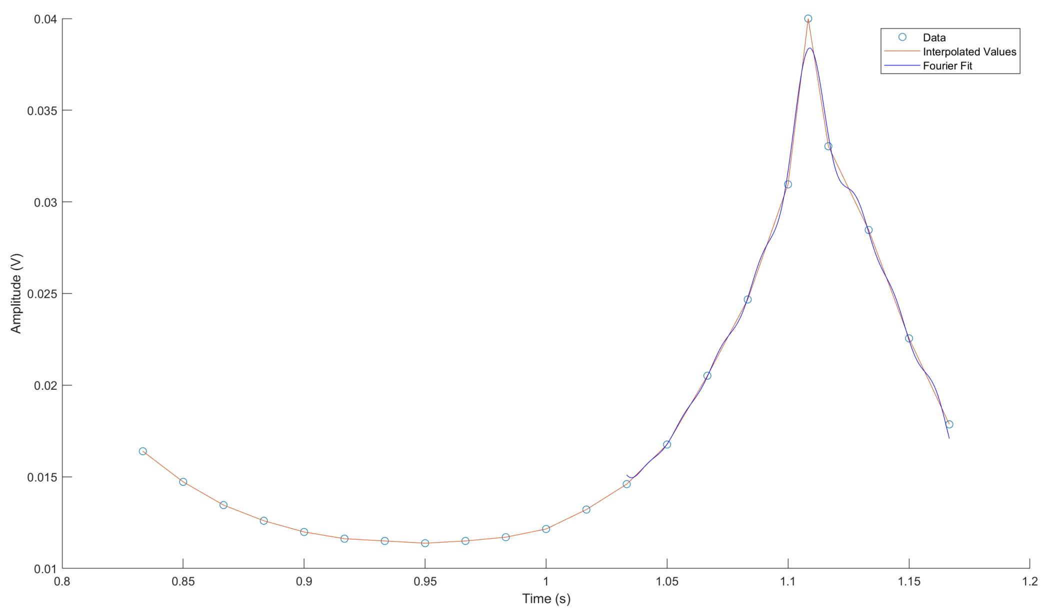

Fig. 5 Fourier fit on fixed frequency data in a synthesized time scale.

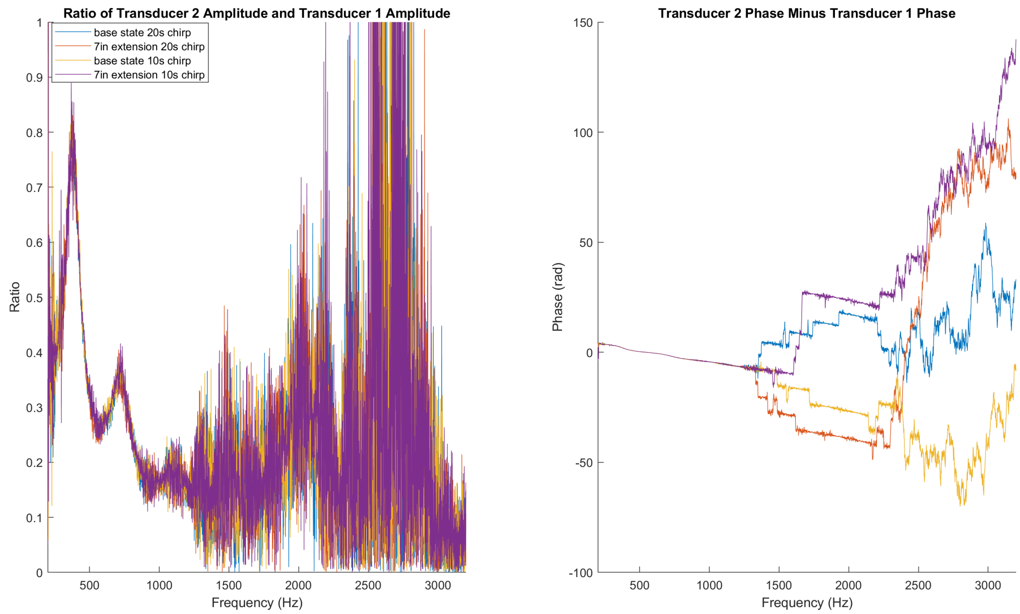

Fig. 6 Transfer function of chirped data for various chirp lengths and tube lengths.

Figure 6 shows the transfer function between four signals found with chirped-frequency measurements. The transfer functions for signals from multiple different chirp durations and tube extension lengths are displayed. As the frequency response of the system increases and the signal-to-noise ratio increases, the transfer function results are heavily overpowered by random noise. For frequencies above 1.2 kHz the signal amplitudes fall into the noise floor of the transducers. Before this point, the transfer functions of all four signals are nearly identical. The same effect causes the phase difference to be highly unpredictable. A potential solution to this problem would be to use microphones instead of pressure transducers for measuring acoustic signals. Pressure transducers were used for this experiment because they would be used in combustion dynamic measurements, but microphones would provide much greater resolution for acoustics.

V. Design and Methodology of an Impedance Experiment

The results of the transfer function experiment indicate that chirped-frequency measurements can accurately match fixed-frequency measurements with an open tube. However, this assumption is invalid to make when performing measurements on a combustion system. With a flow-restricted exit, the acoustic signal will partially reflect from the combustor exit. The pressure recorded in the transducers will be a combination of the incoming and reflected signals and is best characterized by the impedance of the exit. In a chirped experiment there is a possibility that there will be signals of different frequency depending on the characteristic lengths involved. In other words, the reference phase method will be invalid over a region where the local frequency cannot be assumed to be constant.

Dimensional analysis suggests the elements of a combustion dynamics measurement should include the rate of change of frequency, local speed of sound, and the characteristic length of the the measurement. Proper arrangement yields the the chirp number, C_n

$$C_n = \frac{df}{dt}\left(\frac{c}{L}\right)^2, \tag{12}$$

where f is the frequency, c is the sound speed, and L is the characteristic length of the measurement. The chirp number then, is a dimensionless value that describes the homogeneity of the local frequency at a point in a tube. A chirp number significantly less than 1 indicates that throughout the chirp, the frequency in the tube is in quasi-equilibrium and can be assumed to be constant throughout the length of the tube. A number significantly greater than 1 indicates that the local frequency in a measurement cannot be assumed to be the expected value from the chirp, and the simplest ULIA algorithm will not work.

A preliminary step to conducting combustion dynamic's measurements would be to use a chirped frequency analysis on an impedance measurement. The chirp number can be examined in an impedance measurement by modifying the distance from an endwall to the location of the microphones on a tube. Figure 8 shows a tube with sets of microphones positioned at two locations, yielding two different chirp numbers. The procedure for determining the acoustic impedance would be the same in either orientation, however the characteristic length of the chirp number changes with the distance of the microphones from the endwall. If the microphones are close to the endwall, the length will be short. C_n can therefore be increased by moving the microphones closer to the speaker side. As the microphones move closer to the speaker, the frequency of the reflected waves will be different than the incident ones (from the speaker) and the measurement will break down. Another method of increasing the chirp number is by decreasing the local speed of sound. The tube in figure 8 has a valve near the exit. If measurements are performed with a rigid end, the tube can be filled with gases that have a greater molecular mass than air, causing the speed of sound to decrease. Finally, the chirp number can change with the rate of change of frequency, however, the physical capabilities of the function generator and speaker must be carefully considered when modulating the frequency at a high rate of change.

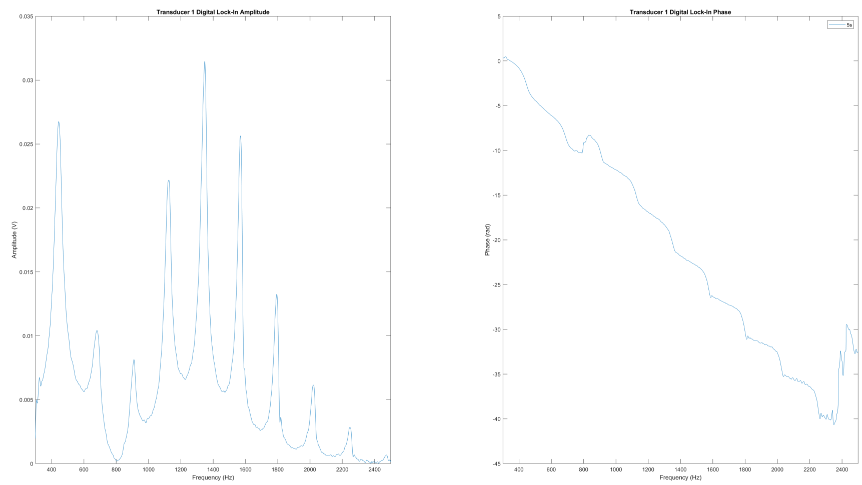

Fig. 7 Frequency response of a chirped-frequency signal in a tube with a covered end.

Figure 7 shows an example of how the results of the ULIA algorithm can change with the a tube closed on both ends.

The procedure outlined by Ďuriš and Labašová was followed to create a tube to characterize the acoustic impedance of a chirped signal [3].

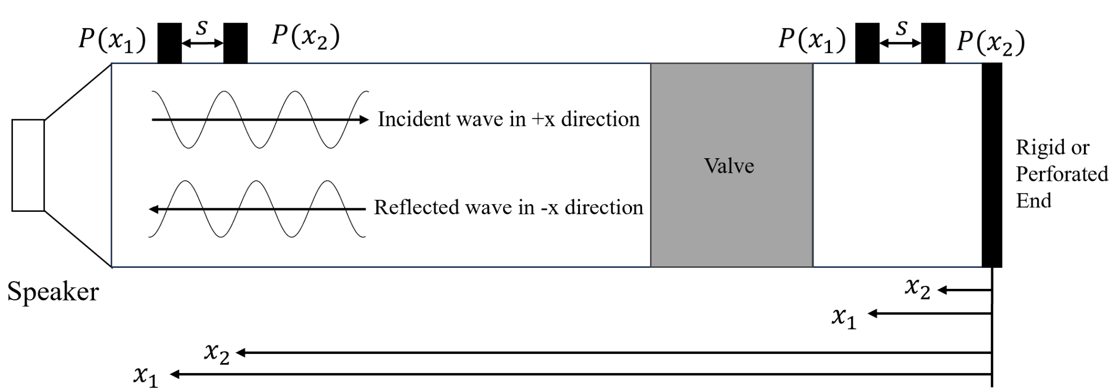

Fig. 8 Scheme of two microphone transfer function method.

Ďuriš and Labašová outline how to determine the acoustic impedance of a signal by measuring the transfer function between the microphones. The total acoustic pressure at any point in the tube is

$$p(x) = p_I(x) + p_R(x) = Ae^{-ikx} + Be^{ikx} \tag{13}$$

p_I is the incident pressure wave, and p_R is the reflected pressure wave. A, B are amplitudes of the pressures in the incident and reflected wave. The transfer function between the two microphones is then

$$H_{12} = \frac{P(x_2, \omega)}{P(x_1, \omega)} = \frac{e^{-ikx_2} + re^{ikx_2}}{e^{-ikx_1} + re^{ikx_1}} \tag{14}$$

Where the acoustic reflection factor r is defined as:

$$r = r(\omega) = \frac{H_{12} - e^{-jks}}{e^{jks} - H_{12}} e^{2jk(s+x_2)} = r_r + jr_i \tag{15}$$

Specific acoustic impedance ratio Z is then calculated from the equation

$$Z = Z(\omega) = \rho c \frac{1+r}{1-r} \tag{16}$$

Z is a complex values function of frequency. Both the real and imaginary components of Z will depend on frequency. A tube was designed to use chirped-frequency signals to measure the acoustic impedance function, but it has not yet been built.

VI. Conclusion

Methods of taking fixed-frequency measurements in a transfer function experiments were discussed. A proposed phase-sensitive method which applies a lock-in amplifier algorithm to a chirped signal was detailed. A common transfer function experiment was performed and both fixed-frequency and chirped-frequency measurements were taken. The chirped-frequency measurements were then processed digitally and results were directly compared to fixed-frequency measurements. The results showed significant similarity with the exception of the peaks of fixed-frequency measurements. A potential solution was proposed to perform experiments with lower rate of frequency change, or slower chirps, to move the information associated with the peaks inside the low pass filter of the ULIA. Chirped-frequency transfer functions are compared across a range of chirp durations and tube lengths and observed to be identical in the region where pressure is finely resolved from the transducers. The gain and phase of the transfer function started to break down near 1200-1300 Hz when the signal to noise levels got too small. A potential solution could be using microphones instead of pressure transducers due to the increased sensitivity of microphones. An experiment analyzing the acoustic impedance of a system was detailed by using a similar tube to a transfer function experiment except with a closed or perforated endwall. The chirp number addition to the impedance experiment was discussed. Chirped-frequency measurements have the potential to provide a near-continuous frequency resolution in a combustion dynamic system. They can save resources such as fuel and labor while providing more data, at the expense of more complex signal processing.

Appendix

Listing 1 Sample ULIA code written in MATLAB

```matlab
function [a, p] = lockin(v, fs, fref, tf)
    % v is the recorded signal, fs is sampling rate
    % fref is reference frequency vector, tf is a time vector
    phase = cumtrapz(tf, fref); % integration of reference time and frequency

    % reference sine and cosine
    refsin = sin(2 .* pi .* phase);
    refcos = cos(2 .* pi .* phase);

    for i = 1:length([60, 120, 180])
```

```
        w0 = freqs(i)/(fs/2); % Normalized frequency
        bw = w0/35; % Bandwidth
        [b, a] = iirnotch(w0, bw);
        v = filtfilt(b, a, v); % notch filter of the signal
    end

    lowpassfreq = 2; % bandpass frequency of the low pass filter

    % x1 signal
    x1 = 2 .* v .* refsin;
    x1 = lowpass(x1, lowpassfreq, fs, Steepness=0.99);
    % y1 signal
    y1 = 2 .* v .* refcos;
    y1 = lowpass(y1, lowpassfreq, fs, Steepness=0.99);
    a = sqrt(x1.^2 + y1.^2); % magnitude of the two signals
    p = atan2(y1,x1); % phase of the two signals
    p = unwrap(p1); % unwrapping the phase
end
```


Acknowledgments

We would like to acknowledge Julia Rivelli from the U.S. Air Force for her contributions to this research.

References

[1] FAUSTI, P., and FARINA, A., "ACOUSTIC MEASUREMENTS IN OPERA HOUSES: COMPARISON BETWEEN DIFFERENT TECHNIQUES AND EQUIPMENT," *Journal of Sound and Vibration*, Vol. 232, No. 1, 2000, pp. 213–229. https://doi.org/https://doi.org/10.1006/jsvi.1999.2694, URL https://www.sciencedirect.com/science/article/pii/S0022460X99926949.

[2] Boyle, D. K., Henderson, B. S., and Hultgren, L. S., "Transfer-Function Determination for Infinite-Tube-Probe Pressure Transducers with Application to Turbofan Core/Combuster Noise," *25th AIAA/CEAS Aeroacoustics Conference*, May 2019. https://doi.org/10.2514/6.2019-2588, URL https://arc.aiaa.org/doi/abs/10.2514/6.2019-2588.

[3] Ďuriš, R., and Labašová, E., "The design of an impedance tube and testing of sound absorption coefficient of selected materials," *IOP Conference Series: Materials Science and Engineering*, Vol. 1050, No. 1, 2021, p. 012003. https://doi.org/10.1088/1757-899X/1050/1/012003, URL https://dx.doi.org/10.1088/1757-899X/1050/1/012003.